Answer to Question #74950, Math / Statistics and Probability

Obtain the maximum likelihood estimate of X in

$$F(y, X) = (1 + X)y^X, 0 < y < 1$$

Solution.

Probability density function:

$$f(y, x_i) = \frac{\partial F(y, X)}{\partial X} = y^{x_i} + (1 + x_i)y^{x_i} \ln y = y^{x_i}(1 + (1 + x_i) \ln y)$$

The likehood function:

$$L(y) = \prod_{i=1}^{n} f(y, x_i) = y^{\sum x_i} \times \prod (1 + (1 + x_i) \ln y)$$

$$\ln L(y) = \ln y \sum x_i + \sum \ln(1 + (1 + x_i) \ln y)$$

We can find y that maximizes $L(y)$ from condition:

$$\frac{\partial \ln L(y)}{\partial y} = \frac{\sum x_i}{y} + \frac{1}{y} \sum \frac{1 + x_i}{1 + (1 + x_i) \ln y} = 0$$

$$\sum x_i + \sum \frac{1 + x_i}{1 + (1 + x_i) \ln y} = 0$$

Solving the last equation in respect of y we get the maximum likelihood estimate $\hat{y}$