Answer on Question #74949-Math-Statistics and Probability

If a random variable U has t- distribution with n degrees of freedom, show that Z=U^2 has the f- distribution with 1 and n degrees of freedom.

Solution

If ξ follows $N(0,1)$ and X follows Chi-square distribution with n degree of freedom are independent, then:

$V = \xi^2$ follows Chi-square with 1 d.f. [Square of standard normal variable] and $U = \frac{\xi}{\sqrt{\frac{X}{n}}}$ follows t-dist with n d.f [being the ratio of standard normal variable to the square root of an independent chi-square variable divided by its d.f]

Thus,

$$Z = U^2 = \left(\frac{\xi}{\sqrt{\frac{X}{n}}}\right)^2 = \frac{\xi^2}{\frac{X}{n}} = \frac{\frac{\xi^2}{1}}{\frac{X}{n}}$$

being the ratio of two independent chi-square variables divided by their respective degree of freedom. Hence, $Z = U^2$ follows F(1,n) [F- distribution with 1 and n degrees of freedom].

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