

Answer to Question #74946, Math / Quantitative Methods

Using finite difference, show that the data $f(-3) = 13, f(-2) = 7, f(-1) = 3, f(0) = 1, f(1) = 1, f(2) = 3, f(3) = 7$. represents a second degree polynomial. obtain this polynomial using interpolation and find $f(2.5)$.

Solution.

We have:

$$\Delta x = 1$$

$$\Delta f_1 = f(-2) - f(-3) = 7 - 13 = -6$$

$$\Delta f_2 = f(-1) - f(-2) = 3 - 7 = -4$$

$$\Delta f_3 = f(0) - f(-1) = 1 - 3 = -2$$

$$\Delta f_4 = f(1) - f(0) = 1 - 1 = 0$$

$$\Delta f_5 = f(2) - f(1) = 3 - 1 = 2$$

$$\Delta f_6 = f(3) - f(2) = 7 - 3 = 4$$

$$\Delta^2 f = \Delta f_2 - \Delta f_1 = \Delta f_3 - \Delta f_2 = \Delta f_4 - \Delta f_3 = \Delta f_5 - \Delta f_4 = \Delta f_6 - \Delta f_5 = 2$$

Since $\Delta^3 f = 0$ then we have second degree polynomial of the form:

$$f(x) = a_2 x^2 + a_1 x + a_0$$

Then:

$$\Delta f = 2a_2 x(\Delta x) + a_2(\Delta x)^2 + a_1(\Delta x)$$

$$\Delta^2 f = 2a_2(\Delta x)^2$$

$$\Delta^2 f = 2a_2 \cdot 1 = 2 \Rightarrow a_2 = 1$$

$$\Delta f_4 = 1 \cdot 1 + a_1 \cdot 1 = 0 \Rightarrow a_1 = -1$$

$$f(0) = a_0 = 1$$

Answer:

$$f(x) = x^2 - x + 1$$
$$f(2.5) = 2.5^2 - 2.5 + 1 = 4.75$$

Answer provided by <https://www.AssignmentExpert.com>