A table of values is to be constructed for the function $f(x) = 1 / (1 + x)$ in the interval $[1,4]$ with equal step length. Determine the spacing h such that quadratic interpolation gives result with accuracy 1×10^{-6}

Solution.

The error is

$$|R_n(x)| \leq \frac{h^3}{9\sqrt{3}} \max_{x\epsilon[a,b]} |f'''(x)|$$

We have:

$$f'(x) = -\frac{1}{(1 + x)^2}$$

$$f''(x) = \frac{2}{(1 + x)^3}$$

$$f'''(x) = -\frac{6}{(1 + x)^4}$$

$$\max_{x\epsilon[1,4]} |f'''(x)| = |f'''(1)| = \left| -\frac{6}{(1 + 1)^4} \right| = \frac{6}{16} = \frac{3}{8}$$

Then:

$$|R_n(x)| \leq \frac{3}{8} \cdot \frac{h^3}{9\sqrt{3}}$$

$$\frac{3}{8} \cdot \frac{h^3}{9\sqrt{3}} = 10^{-6}$$

Answer:

$$h = \sqrt[3]{\frac{8 \cdot 9\sqrt{3}}{3} \cdot 10^{-6}} = \frac{2}{100} \sqrt[3]{3\sqrt{3}} = 0.03464$$