Answer on Question # 74943, Math-Differential Equations:

Question: Solve the following ordinary differential equation:

(a) $\frac{dy}{dx} = \frac{y-x}{x-4y}$

(b) $(2yx^2+4)\frac{dy}{dx} + (2y^2x-3)=0$

(c) $y''+3y'-10y=3x^2$

Solution: (a). $\frac{dy}{dx} = \frac{y-x}{x-4y}$

This equation is exact equation i.e. write in the form M dx + N dy = 0

M = y - x and N = x – 4y

So, $\Psi = \int (N\,dy) = xy - 2y^2 + C$ (C is integration constant)

Now replace C with m(x) , as x was treated as a constant.

$\Psi = xy - 2y^2 + m(x)$

Now compare the value of $\frac{\partial}{\partial x}\left(xy - 2y^2 + m(x)\right)\ and\ (y-x)$

So, $\frac{\partial}{\partial x}\left(xy - 2y^2 + m(x)\right) = (y-x)$

m(x) = D (constant)

So, $\Psi = xy - 2y^2 + D$

(b). $(2yx^2+4)\frac{dy}{dx} + (2y^2x-3) =0$

This equation is in exact form i.e. $M(x, y) + N(x, y)\frac{dy}{dx} = 0$

Here, $M(x, y) = 2y^2x-3$ and $N(x, y) = 2yx^2+4$

$\Psi = \int(N\,dy) = 4y + x^2y^2 + c$ (c = integration constant)

Now replace c with m(x) , as x was treated as a constant.

So, $\Psi = 4y + x^2y^2 + m(x)$

Now compare the value of $\frac{\partial}{\partial x}\left(4y + x^2y^2 + m(x)\right) = 2x\,y^2 - 3$

So we get, $m(x) = -3x + c_1$ (c_1 is another constant)

So, $\Psi = 4y + x^2y^2 -3x + c_1$

(c). y''+3y'-10y=3x^2

To find complementary solution , we put y''+3y'-10y = 0 ……………………(1)

Solution of equation (1) becomes, $y = Ce^{2x}$+ D e^{-5x}

Now particular solution is z = $-3\frac{x^2}{10} - \frac{9x}{50} - \frac{57}{500}$

So, the total solution is Ψ = y + z = Ce^{2x}+ D e^{-5x} $-3\frac{x^2}{10} - \frac{9x}{50} - \frac{57}{500}$

Where C and D are constants.

Answer: So, the answers are (a). Ψ = xy – 2y^2 + D , (b). Ψ = 4y + x^2y^2 -3x + c_1 ,
(c). Ψ= Ce^{2x}+ D e^{-5x} $-3\frac{x^2}{10} - \frac{9x}{50} - \frac{57}{500}$.