

Answer on question #74912:

The area of the figure under the curve $\begin{cases} x = 2t - 1 \\ y = t^2 + 5 \end{cases}$, $-4 \leq t \leq 4$ and up to X-axis is equal to $165\frac{1}{3}$.

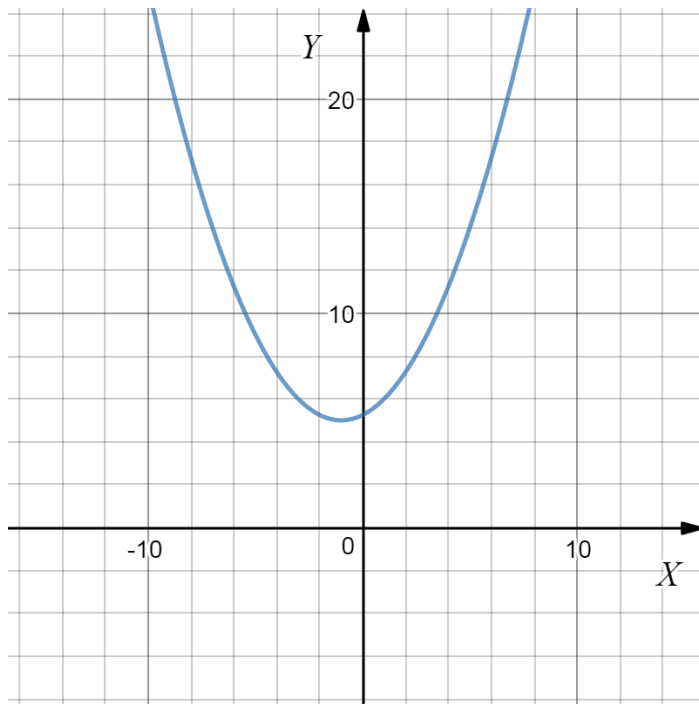
$$\begin{cases} x = 2t - 1 \\ y = t^2 + 5 \end{cases}, -4 \leq t \leq 4.$$

- 1) Our function is given in parametric form.
 - a) $\min_{t \in [-4;4]} x = x(-4) = -9$;
 $\max_{t \in [-4;4]} x = x(4) = 7$.
So that the domain of our function is $[-9,7]$.
 - b) $y = t^2 + 5$ so that $y \geq 5$, so that the codomain of our function is $[5; +\infty]$.

- 2) The graph of our function intersects Y-axis at
 $x = 2t - 1 = 0$;
 $t = \frac{1}{2}$.
The graph of our function doesn't intersect X-axis because $y \geq 5$.

- 3) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{2t}{2} = t$;
There is local minimum point at $t = 0$:
 $x_{min} = -1$; $y_{min} = 5$.
Except that $\min_{[-9;7]} y = 5$.

- 4) $\frac{d^2y}{dx^2} = \frac{1}{2} > 0$. So that our function is strict convex.



5)

- 6) The area of the figure under the curve $\begin{cases} x = 2t - 1 \\ y = t^2 + 5 \end{cases}, -4 \leq t \leq 4$ and up to X-axis

$$S = \int_{-9}^7 y \cdot dx = [x = 2t - 1; dx = 2dt; t \in [-4; 4]] = 2 \int_{-4}^4 (t^2 + 5) dt = 4 \int_0^4 (t^2 + 5) dt = 4 \left(\frac{t^3}{3} + 5t \right) \Big|_0^4 = 4 \left(\frac{64}{3} + 20 \right) = 165 \frac{1}{3}.$$

Answer provided by <https://www.AssignmentExpert.com>