Answer on Question #74856 – Math – Analytic Geometry

Question

$\overline{A}$=5i+7j+8k
$\overline{B}$= 5i+2j-5k

Use your understanding of vector analysis to complete the following:

1. Calculate a resultant vector, which would theoretically represent a single force that could replace the two force vectors A and B while giving the same support to the structure.
2. Calculate the modulus of all three forces.
3. Determine the value of the dot-product (scalar product) of vectors A and B
4. Calculate the angle between the vectors A and B
5. Determine the directional cosine angles of the resultant vector with respect to the x, y and z axes.

Solution

1. Resultant vector (C) of vectors A and B is the sum of those vectors:

$$\overline{C} = \overline{A} + \overline{B} = (5i + 7j + 8k) + (5i + 2j - 5k) = (5 + 5)i + (7 + 2)j + (8 - 5)k$$
$$= 10i + 9j + 3k$$

2. Modulus of the vector A:

$$\left|\overline{A}\right| = \sqrt{5^2 + 7^2 + 8^2} = \sqrt{25 + 49 + 64} = \sqrt{138}$$

Modulus of the vector B:

$$\left|\overline{B}\right| = \sqrt{5^2 + 2^2 + (-5)^2} = \sqrt{25 + 4 + 25} = \sqrt{54} = 3\sqrt{6}$$

Modulus of the vector C:

$$\left|\overline{C}\right| = \sqrt{10^2 + 9^2 + 3^2} = \sqrt{100 + 81 + 9} = \sqrt{190}$$

3. Scalar product of vectors A and B:

$$\overline{A} \cdot \overline{B} = 5 \cdot 5 + 7 \cdot 2 + 8 \cdot (-5) = 25 + 14 - 40 = -1$$

4. Cosine of the angle between vectors A and B (θ):

$$\cos\theta = \frac{\overline{A} \cdot \overline{B}}{\left|\overline{A}\right| \cdot \left|\overline{B}\right|} = \frac{-1}{\sqrt{138} \cdot 3\sqrt{6}} = \frac{-1}{3\sqrt{828}} = \frac{-1}{18\sqrt{23}}$$

Angle between vectors A and B (θ):

$$\theta = \cos^{-1}(\cos\theta) = \cos^{-1}\frac{-1}{18\sqrt{23}} = 1.58238072$$

5. Directional cosine angle of the vector C with respect to the x axis:

$$\cos\alpha = \frac{10}{|\overline{C}|} = \frac{10}{\sqrt{190}}$$

Directional cosine angle of the vector C with respect to the y axis:

$$\cos\beta = \frac{9}{|\overline{C}|} = \frac{9}{\sqrt{190}}$$

Directional cosine angle of the vector C with respect to the z axis:

$$\cos\gamma = \frac{3}{|\overline{C}|} = \frac{3}{\sqrt{190}}$$

Answer:

1. Resultant vector of vectors A and B:

$$\overline{C} = 10i + 9j + 3k$$

2. Modulus of the vector A:

$$|\overline{A}| = \sqrt{138}$$

Modulus of the vector B:

$$|\overline{B}| = 3\sqrt{6}$$

Modulus of the vector C:

$$|\overline{C}| = \sqrt{190}$$

3. Scalar product of vectors A and B:

$$\overline{A} \cdot \overline{B} = -1$$

4. Angle between vectors A and B:

$$\theta = \cos^{-1}\frac{-1}{18\sqrt{23}} = 1.58238072$$

5. Directional cosine angles of the resultant vector C:

with respect to the x axis:

$$\cos\alpha = \frac{10}{|\overline{C}|} = \frac{10}{\sqrt{190}}$$

with respect to the y axis:

$$\cos\beta = \frac{9}{|\overline{C}|} = \frac{9}{\sqrt{190}}$$

with respect to the z axis:

$$\cos\gamma = \frac{3}{|\overline{C}|} = \frac{3}{\sqrt{190}}$$