Answer on Question #74854 - Math - Calculus

Question

Express the vector field vector $F = xzi + (x^2 + y^2)j + \frac{x}{z}k$ in cylindrical polar coordinates.

Solution

Any vector field can be written as

$$F = F_x i + F_y j + F_z k = F_\rho e_\rho + F_\varphi e_\varphi + F_z e_z,$$

for our condition

$$F_x = xz, F_y = x^2 + y^2, F_z = \frac{x}{z}.$$

Use the next conversion between cartesian and cylindrical coordinates:

 $x = \rho cos \varphi, y = \rho sin \varphi, z = z.$

Let's substitute in the condition:

$$F_{\rho} = xz = \rho \cos\varphi \cdot z = \rho z \cos\varphi,$$

$$F_{\varphi} = x^{2} + y^{2} = (\rho^{2} \cos^{2}\varphi + \rho^{2} \sin^{2}\varphi) = \rho^{2},$$

$$F_{z} = \frac{x}{z} = \frac{\rho \cos\varphi}{z}.$$

Then

$$F = \rho z \cos\varphi \, e_{\rho} + \rho^2 e_{\varphi} + \frac{\rho \cos\varphi}{z} e_z.$$

Answer: $F = \rho z \cos \varphi \ e_{\rho} + \rho^2 e_{\varphi} + \frac{\rho \cos \varphi}{z} e_z.$