Answer on Question #74823 – Math – Calculus

Question

Show that the function u(x,t)=e^(-6t)cos2x is a solution of the one dimensional heat equation

Solution

Function $u(x,t) = e^{-6t}\cos 2x$.

One-dimensional heat equation:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$

where k is called thermal diffusivity. One can find partial derivatives:

$$\frac{\partial u}{\partial t} = -6e^{-6t}\cos 2x\,;$$

$$\frac{\partial u}{\partial x} = -2e^{-6t}\sin 2x\,;$$

$$\frac{\partial^2 u}{\partial x^2} = -4e^{-6t}\cos 2x\,;$$

$$-6e^{-6t}\cos 2x = k\cdot(-4e^{-6t}\cos 2x);$$

$$k = 3/2.$$

So we can see that the function $u(x,t)$ is indeed a solution of the one-dimensional heat equation with thermal diffusivity $k = 3/2$.