Answer on Question #74731 – Math – Calculus

Question

If xsiny=sin(p+y), p belongs to R, show that sinp.dy/dx + sin^2y=0

Solution

We need to show that $\sin p\frac{dy}{dx}+\sin^2 y=0$ if $x\sin y=\sin(p+y),\ \ p\in\mathbb{R}$

We rewrite the condition in the form

$$F(x,y)=x\sin y-\sin(p+y)=0$$

This equation defines y implicitly as a function of x.

The implicit differentiation with respect to x gives

$$\frac{dy}{dx}=-\frac{F_x(x,y)}{F_y(x,y)}$$

Find $F_x(x,y)$ and $F_y(x,y)$

$$F_x(x,y)=\sin y$$

$$F_y(x,y)=x\cos y-\cos(p+y)$$

Then we get

$$\frac{dy}{dx}=-\frac{\sin y}{x\cos y-\cos(p+y)}$$

or

$$\left(x\cos y-\cos(p+y)\right)\frac{dy}{dx}+\sin y=0$$

Multiplying both sides of this equation by $\sin y$ we get

$$(x\cos y\sin y-\cos(p+y)\sin y)\frac{dy}{dx}+\sin^2 y=0$$

Take into account that $x\sin y=\sin(p+y)$

$$(\sin(p+y)\cos y\ -\cos(p+y)\sin y)\frac{dy}{dx}+\sin^2 y=0$$

Using the formula

$$\sin\alpha\cos\beta-\cos\alpha\sin\beta=\sin(\alpha-\beta)$$

we get

$$\sin(p+y)\cos y\ -\cos(p+y)\sin y=\sin[(p+y)-y]=\sin p$$

and finally

$$\sin p\frac{dy}{dx}+\sin^2 y=0$$

Answer: $\sin p\frac{dy}{dx}+\sin^2 y=0$ if $x\sin y=\sin(p+y),\ \ p\in\mathbb{R}$.