

## Answer on Question #74722 – Math – Calculus

### Question

Find the area enclosed by the curve  $r = a(1 - \cos \theta)$

### Solution

The unknown area is expressed this way:  $S = \frac{1}{2} \int_0^{2\pi} r^2 d\theta$

$$\text{That is } S = \frac{1}{2} \int_0^{2\pi} a^2 (1 - \cos \theta)^2 d\theta = \frac{a^2}{2} \int_0^{2\pi} (1 - 2 \cos \theta + \cos^2 \theta) d\theta$$

Remembering that  $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ , we have:

$$\begin{aligned} S &= \frac{a^2}{2} \int_0^{2\pi} \left( 1 - 2 \cos \theta + \frac{1}{2} + \frac{1}{2} \cos 2\theta \right) d\theta = \\ &= \frac{a^2}{2} \left( \int_0^{2\pi} \frac{3}{2} d\theta - \int_0^{2\pi} 2 \cos \theta d\theta + \int_0^{2\pi} \frac{1}{2} \cos 2\theta d\theta \right) = \\ &= \frac{a^2}{2} \left( \frac{3}{2} (2\pi - 0) - 2(\sin 2\pi - \sin 0) + \frac{1}{4} (\sin 4\pi - \sin 0) \right) = \\ &= \frac{a^2}{2} (3\pi - 0 + 0) = \frac{3}{2} \pi a^2 \end{aligned}$$

**Answer:**  $S = \frac{3}{2} \pi a^2$ .