Answer on Question #74720, Math / Calculus

If $x \sin y = \sin(p+y), p \in \mathbb{R}$, show that $\sin p \, dy/dx + \sin^2 y = 0$.

Solution

$$x \sin y = \sin(p+y)$$

Differentiate the both sides of the equation with respect to x using the Product Rule and Chain Rule

$$\sin y + x \cos y \frac{dy}{dx} = \cos(p+y) \frac{dy}{dx}$$

Substitute

$$x = \frac{\sin(p+y)}{\sin y}$$

$$\sin y + \frac{\sin(p+y)}{\sin y} \cos y \frac{dy}{dx} = \cos(p+y) \frac{dy}{dx}$$

$$\sin y + \left(\frac{\sin(p+y)}{\sin y} \cos y - \cos(p+y) \right) \frac{dy}{dx} = 0$$

$$\sin^2 y + (\sin(p+y) \cos y - \sin y \cos(p+y)) \frac{dy}{dx} = 0$$

Use the Sine – Difference formula

$$\sin p = \sin((p+y) - y) = \sin(p+y) \cos y - \sin y \cos(p+y)$$

Then

$$\sin^2 y + \sin p \frac{dy}{dx} = 0$$

Therefore

$$\sin p \frac{dy}{dx} + \sin^2 y = 0, \text{if } x \sin y = \sin(p+y), p \in \mathbb{R}.$$