Answer on Question #74645 – Math – Calculus

Question

Examine whether lim x to 0 (e^1/x)/(e^1/x +1) exist or not..

Solution

We need to determine the existence of limit

$$\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x}+1}$$

Since x can tends to both 0^+ and 0^- we will examine both cases.

Let $\frac{1}{x}=y$. If x tends towards 0^+, then y tends towards $+\infty$ and we have

$$\lim_{x\to 0^+}\frac{e^{1/x}}{e^{1/x}+1}=\lim_{y\to+\infty}\frac{e^y}{e^y+1}=\lim_{y\to+\infty}\frac{e^y}{e^y(1+e^{-y})}=\lim_{y\to+\infty}\frac{1}{1+e^{-y}}=\frac{1}{1+e^{-\infty}}=\frac{1}{1+0}=1$$

Let $\frac{1}{x}=y$. If x tends towards 0^-, then y tends towards $-\infty$ and we have

$$\lim_{x\to 0^-}\frac{e^{1/x}}{e^{1/x}+1}=\lim_{y\to-\infty}\frac{e^y}{e^y+1}=\lim_{y\to-\infty}\frac{e^y}{e^y(1+e^{-y})}=\lim_{y\to-\infty}\frac{1}{1+e^{-y}}=\frac{1}{1+e^{\infty}}=\frac{1}{1+\infty}=0$$

Thus, the one-sided limits are not equal and therefore the limit

$$\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x}+1}$$

does not exist.

Answer: the limit

$$\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x}+1}$$

does not exist.