Answer on Question #74644 – Math – Calculus

Question

State whether the following statements are true or false. Justify your answers.
1) lim x to 0 (1/x^2 -1/sin^2x)is in (0/0) form.
2) f(x,y)={sin(x^2y/x^3 +y^3)}/ln(x+y/x) is a homogeneous funtion of degree 2.
3) Domain of f(x,y)=xy/(x^4 +y^4) is R^2.
4) The function f(x,y)=(x^3y +1, x^2 +y^2) is locally invertible at (1,2).
5) The function f(x,y)=x^3 +y^3 is integrable on [1,2]×[1,3].

Solution

1) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)$ $is\ in \left(\frac{0}{0}\right)\ form$. It is false.

$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)$ $is\ in\ (\infty-\infty)\ form$ but

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)=\lim_{x\to 0}\left(\frac{sin^2x-x^2}{x^2sin^2x}\right)=\left(\frac{0}{0}\right)$$

$$=[we\ can\ use\ l'Hospitale\ rule]=\lim_{x\to 0}\frac{(sin^2x-x^2)'}{(x^2sin^2x)'}=$$

$$=\lim_{x\to 0}\frac{2\sin(2x)-2x}{2\sin(2x)\cdot x^2+2x\cdot sin^2x}=\left(\frac{0}{0}\right)=[we\ can\ use\ l'Hospitale\ rule]$$

$$=$$

$$=\lim_{x\to 0}\frac{4\cos(2x)-2}{4\cos(2x)\cdot x^2+2\sin(2x)\cdot 2x+2\,sin^2x+2x\cdot 2sin(2x)}=\frac{2}{0}=\infty$$

2) $f(x,y)=\frac{sin\left(\frac{x^2y}{x^3+y^3}\right)}{ln\left(x+\frac{y}{x}\right)}$ is a homogeneous function of degree 2. It is false.

$$f(\alpha x,\alpha y)=\frac{sin\left(\frac{(\alpha x)^2\alpha y}{(\alpha x)^3+(\alpha y)^3}\right)}{ln\left(\alpha x+\frac{\alpha y}{\alpha x}\right)}=\frac{sin\left(\frac{\alpha^3x^2y}{(x^3+y^3)\alpha^3}\right)}{ln\left(\alpha x+\frac{y}{x}\right)}=$$

$$=\frac{sin\left(\frac{x^2y}{x^3+y^3}\right)}{ln\left(\alpha x+\frac{y}{x}\right)}\neq\alpha^2 f(x,y)$$

3) Domain of $f(x,y)=\frac{xy}{x^4+y^4}$ is R^2. It is false. $x^4+y^4\neq 0$. Domain of $f(x,y)$ is $R^2/(0;0)$.

4) The function $f(x,y)=\begin{bmatrix}x^3y+1\\x^2+y^2\end{bmatrix}$ is locally invertible at (1,2). This is true.

$$\text{Jacobian matrix of f } J_{f(x,y)}=\begin{bmatrix}\frac{\partial f}{\partial x}&\frac{\partial f}{\partial y}\\\frac{\partial f}{\partial x}&\frac{\partial f}{\partial y}\end{bmatrix}=\begin{bmatrix}3x^2y&x^3\\2x&2y\end{bmatrix}$$

$$det\left(J_{f(x,y)}\right) = \begin{vmatrix} 3x^2y & x^3 \\ 2x & 2y \end{vmatrix} = 6x^2y^2 - 2x^4$$

$$det\left(J_{f(x,y)}\right)\Big|_{\substack{x=1 \\ y=2}} = 24 - 2 = 22 \neq 0$$

5) The function $f(x, y) = x^3 + y^3$ is integrable on $[1; 2] \times [1; 3]$. Of course this is true because x^3 is continuous function on R, y^3 is continuous function on R, so $f(x, y)$ is continuous function on R^2 and that's why it is integrable on $[1; 2] \times [1; 3]$.

Answer:

1) false;

2) false;

3) false;

4) true;

5) true.