Find the equation of circle with center at the origin and tangent to the line $2x - 5y = 8$.

Solution:

Equation of circle with center at the origin $(0,0)$ and radius r: $\quad x^2 + y^2 = r^2, \quad r > 0.$

Points of intersection of the circle and a line:

$$\begin{cases} 2x - 5y = 8 \\ x^2 + y^2 = r^2 \end{cases}$$

$$\begin{cases} x = \frac{1}{2}(5y + 8) \\ x^2 + y^2 = r^2 \end{cases}$$

$$\frac{1}{4}(5y + 8)^2 + y^2 = r^2$$

$$25y^2 + 80y + 64 + 4y^2 - 4r^2 = 0$$

$$29y^2 + 80y + (64 - 4r^2) = 0$$

$$D = 80^2 - 4 \cdot 29 \cdot (64 - 4r^2) = 6400 - 7424 + 464r^2 = 464r^2 - 1024$$

Circle is tangent to line $\Longleftrightarrow$ Circle and line have only one intersection $\Longleftrightarrow D = 0$.

$$D = 464r^2 - 1024 = 0$$

$$r^2 = \frac{1024}{464} = \frac{64}{29}$$

Equation of circle: $\quad x^2 + y^2 = r^2 = \frac{64}{29}$

Answer: $\quad x^2 + y^2 = \frac{64}{29}$