Answer on Question #74604 – Math – Algebra

Question

Write an odd natural number as a sum of two integers m_1 and m_2 in a way that $m_1 m_2$ is maximum.

Solution

Consider the natural odd number $2m+1, m$ is given.

Let $m_1 + m_2 = 2m + 1, m_1 \in \{0, 1, \dots 2m+1\}, m_2 \in \{0, 1, \dots 2m+1\}$.

Then $m_2 = 2m + 1 - m_1$.

Consider the product $m_1 m_2$ as the function $f(m_1)$

$f(m_1) = m_1(2m + 1 - m_1)$

Take the first derivative with respect to m_1

$$f'(m_1) = \left(m_1(2m+1-m_1)\right)' = 2m + 1 - m_1 + m_1(0 + 0 - 1) =$$
$$= 2m + 1 - 2m_1$$

Find the critical point(s)

$$f'(m_1) = 0 => 2m + 1 - 2m_1 = 0 => m_1 = m + \frac{1}{2}$$

$$If\ 0 \le m_1 < m + \frac{1}{2}, f'(m_1) > 0, f(m_1)\ increases$$

$$If\ m + \frac{1}{2} < m_1 \le m, f'(m_1) < 0, f(m_1)\ decreases$$

The function $f(m_1)$ has maximum at $m_1 = m + 1/2$.

Since m_1 is integer number, then we consider

$m_1 = m + 0$ or $m_1 = m + 1$

Therefore, we have that

$2m + 1 = (m) + (m + 1)$

If M is the odd natural number, then

$$M = \frac{1}{2}(M-1) + \frac{1}{2}(M+1)$$

The maximum of product

$$\frac{1}{2}(M-1)\left(\frac{1}{2}\right)(M+1) = \frac{M^2 - 1}{4}$$