Answer on Question #74598 – Math – Linear Algebra

Question

Give example, with justification , of the following:

(1) two non-zero , 3×3 matrices A and B , with $|A| = 0, |B| = \frac{5}{7} i$;

Solution

Let's consider the following non-zero matrices (all elements of zero- matrix are zeroes)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 2 \\ 0 & \frac{i}{7} & 1 \end{pmatrix}.$$

$|A| = \det(A) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0$ because the matrix A has linearly dependent rows: $(2; 2; 2) = 2 \cdot (1; 1; 1)$.

$$|B| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & i & 2 \\ 0 & \frac{i}{7} & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} i & 2 \\ \frac{i}{7} & 1 \end{vmatrix} = i - \frac{2i}{7} = i\left(1 - \frac{2}{7}\right) = \frac{5}{7} i \, .$$

Question

Give example, with justification , of the following:

(2) two non–singular 2×2 matrices C and D , with $|C| = \sqrt{2} \cdot |D|$.

Solution

Let's consider the following non–singular matrices (the determinant of a singular matrix is equal to zero)

$D = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, C = \begin{pmatrix} 2\sqrt{2} & 1 \\ \sqrt{2} & 1 \end{pmatrix}.$

$|D| = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1; \; |C| = \begin{vmatrix} 2\sqrt{2} & 1 \\ \sqrt{2} & 1 \end{vmatrix} = 2\sqrt{2} - \sqrt{2} = \sqrt{2};$

So $|C| = \sqrt{2} \cdot |D|$.

Answer:

Example (1): $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 2 \\ 0 & \frac{i}{7} & 1 \end{pmatrix}.$

Example (2): $D = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 2\sqrt{2} & 1 \\ \sqrt{2} & 1 \end{pmatrix}$.