Answer on Question #74588 - Math / Complex Analysis - for completion.

Obtain the polar and exponential representations of z_1, z_2 and $z_1 z_2$, where $z_1 = \frac{1}{2} - 2i$ and $z_2 = 3 + i$.

Solution:

$$z_1 = \frac{1}{2} - 2i;$$

$|z_1| = \sqrt{x^2 + y^2}$, where $x = Re\, z_1 = \frac{1}{2}, y = Im\, z_1 = -2;$

$$arg\, z_1 = \varphi;\ \varphi = arc\tan\frac{y}{x},$$

$$|z_1| = \sqrt{\left(\frac{1}{2}\right)^2 + (-2)^2} = \sqrt{\frac{1}{4} + 4} = \sqrt{\frac{1+16}{4}} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2} \approx 2.0616;$$

$$\varphi = arc\tan\frac{-2}{\frac{1}{2}} = arc\tan(-4);$$

Polar and exponential representation of z_1:

$$z_1 = \frac{\sqrt{17}}{2}(\cos(arc\tan(-4)) + isin(arc\tan(-4)));$$

$$z_1 = \frac{\sqrt{17}}{2}e^{iarc\tan(-4)};$$

$$z_2 = 3 + i;$$

$|z_2| = \sqrt{x^2 + y^2}$, where $x = Re\, z_2 = 3, y = Im\, z_2 = 1;$

$$arg\, z_2 = \varphi;\ \varphi = arc\tan\frac{y}{x},$$

$$|z_2| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10} \approx 3.1623;$$

$$\varphi = arc\tan\frac{1}{3};$$

Polar and exponential representation of z_2:

$$z_2 = \sqrt{10}\left(\cos\left(arc\tan\frac{1}{3}\right) + isin(arc\tan\frac{1}{3}\right));$$

$$z_2 = \sqrt{10}e^{iarc\tan\frac{1}{3}};$$

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(y_1x_2 + x_1y_2);$$

$$z_1z_2 = \left(\frac{1}{2} - 2i\right)(3+i) = \left(\frac{1}{2} * 3 — 2\right) * 1 + i\left((-2) * 3 + \frac{1}{2} * 1\right) = \left(\frac{3}{2} + 2\right) + i\left(-6 + \frac{1}{2}\right)$$
$$= \frac{7}{2} - i\frac{11}{2};$$

Let $z_3 = z_1z_2$;

$$|z_3| = \sqrt{\left(\frac{7}{2}\right)^2 + \left(\frac{-11}{2}\right)^2} = \sqrt{\frac{49}{4} + \frac{121}{4}} = \sqrt{\frac{170}{4}} = \frac{\sqrt{170}}{2} \approx 6.5192;$$

$$\varphi = arc\tan\frac{\frac{-11}{2}}{\frac{7}{2}} = arc\tan\left(-\frac{11}{7}\right);$$

Polar and exponential representation of z_1z_2:

$$z_1z_2 = \frac{\sqrt{170}}{2}\left(\cos\left(arc\tan\left(-\frac{11}{7}\right)\right) + isin(arc\tan\left(-\frac{11}{7}\right)\right));$$

$$z_1z_2 = \frac{\sqrt{170}}{2}e^{iarc\tan\left(-\frac{11}{7}\right)}.$$