Answer on Question #74549 – Math – Discrete Mathematics

Question

An integer solution to the equation 3x+4=7y is an ordered pair of integers (x, y) that satisfies the equation. For example, (1,1) is one such solution. Write the set of all integer solutions to the equation 3x + 4 = 7y in set builder notation.

Solution

$3x + 4 = 7y$

$3x = 7y - 4$

Residue method

The left and the right side of the equation are divided into 3 groups.

if $y = 3m, \ m \in Z$ then $7y - 4 = 7(3m) - 4 = 21m - 4$ $\quad (21m - 4) \ mod \ 3 \neq 0$

if $y = 3m + 1, \ m \in Z$ then $7y - 4 = 7(3m + 1) - 4 = 21m + 3$ $\quad (21m + 3) \ mod \ 3 = 0$

if $y = 3m + 2, \ m \in Z$ then $7y - 4 = 7(3m + 2) - 4 = 21m + 10$ $(21m + 10) \ mod \ 3 \neq 0$

$y = 3m + 1, \qquad m \in Z$

$3x = 7y - 4 = 7(3m + 1) - 4 = 21m + 3$

$x = 7m + 1$

Answer:

$\{(x; y) | x = 1 + 7m, y = 1 + 3m, m \in Z\}$