Question #7452 Solve $1 - x(x + y)dy/dx = y(x - y)$.
Solution. The equation you offered does not belong to any "good" class of the equation, that could solved by some standard methods. You might misprinted and the real eqaution is $x(x + y)dy/dx = y(x - y)$. This equation is homogeneous(it does not change when we make the substitution $x \mapsto \lambda x\,, y \mapsto \lambda y$). Hence, it is reasonable to make a substitution $y = zx$, $y' = z'x + z$, thus our equation is equivalent to

$$\frac{x}{x + xz}(z'x + z) = \frac{zx}{x - zx}, \text{ or } \frac{1}{1 + z}(z'x + z) = \frac{z}{1 - z}.$$

We get $z' \cdot x = \frac{z(1 + z)}{1 - z} - z = \frac{z + z^2 - z + z^2}{1 - z}$, finally we obtain $1/2dz\frac{1 - z}{z^2} = dx/x$. Or $1/2(-1/z - \log|z|) = \log|x| + C$, returning back to initial variables $1/2(-\frac{x}{y} + \log|x| - \log|y|) = \log|x| + C$.
Answer. The general solution can be obtained from the relation $-\frac{x}{y} - \log|y| = \log|x| + C$, $C \in \mathbb{R}$ is arbitrary real constant.