Answer on Question #74429 – Math – Real Analysis

Question

Let $\{b_n\}$ be a sequence such that $b_n = 1 + \frac{1}{n}$. Prove that $inf(b_n) = 1$

Solution

1) Since $\frac{1}{n} > 0$ all $b_n > 1$ and hence $inf(b_n) \geq 1$
2) For any $\epsilon > 0$ let us take $N_\epsilon = [\frac{1}{\epsilon}] + 1$. Since $N_\epsilon > \frac{1}{\epsilon}$, $\frac{1}{N_\epsilon} < \epsilon$ and $b_{N_\epsilon} < 1 + \epsilon$. Hence for any $\epsilon > 0$, $inf(b_n) \leq b_{N_\epsilon} < 1 + \epsilon$. Therefore $inf(b_n) \leq 1$.

Since $inf(b_n) \geq 1$ and $inf(b_n) \leq 1$, $inf(b_n) = 1$.