Question #74300, Math / Differential Equations

Identify the following differential equations and hence solve them

1) $y' = -\frac{4}{x^2} - \frac{y}{x} + y^2$
2) $y = xy' + 1 - \ln y'$

1.

Solution

$$y' = -\frac{4}{x^2} - \frac{y}{x} + y^2$$

This is the first-order nonlinear ordinary differential equation

$$x^2y' = -4 - xy + x^2y^2$$

$$x^2y^2 = x^2y' + xy + 4$$

$$x^2y^2 = x(xy' + y) + 4$$

$$xy' + y = (xy)'$$

$$(xy)^2 = x(xy)' + 4$$

Substitute $z = xy$

$$z^2 = xz' + 4$$

$$xz' = z^2 - 4$$

$$\frac{dz}{z^2 - 4} = \frac{dx}{x}$$

$$\int \frac{dz}{z^2 - 4} = \int \frac{dx}{x}$$

$$\frac{1}{4}\ln\left(\frac{z-2}{z+2}\right) = \ln x + C$$

$$\ln\left(\frac{z-2}{z+2}\right) = \ln x^4 + C$$

$$\frac{z-2}{z+2} = C\,x^4$$

$$z - 2 = C\,x^4(z+2)$$

$$z(1 - C\,x^4) = 2C\,x^4 + 2$$

$$z = \frac{2C\,x^4 + 2}{1 - C\,x^4}$$

$$Get\ back\ to\ substitute$$

$$xy = \frac{2C\,x^4 + 2}{1 - C\,x^4}$$

$$y = \frac{C\,x^4 + 2}{x(1 - C\,x^4)}$$

Answer:

$$y = \frac{C\,x^4 + 2}{x(1 - C\,x^4)}$$

2.

Solution

$$y = \ xy' + 1 - \ln y'$$

This is the Clero equation of the form $y \ = \ x\,y\,' + \ f\ (y')$. Parametrizize

$$\left[\begin{array}{c} \begin{cases} y' = p \\ x = x \\ y = xp + 1 - \ln p \end{cases} \\ dy = y'dx \end{array}\right.$$

$$xdp + pdx - \frac{dp}{p} = pdx$$

$$dp(x - \frac{1}{p}) = 0$$

$$dp = 0 \qquad\qquad x - \frac{1}{p} = 0$$

$$p = C \qquad\qquad p = \frac{1}{x}$$

$$y = Cx + 1 - \ln C \qquad\qquad y = 2 + lnx$$

Answer:

$$\left[\begin{array}{l} y = Cx + 1 - \ln C \\ y = 2 + lnx \end{array}\right.$$

Answer provided by https://www.AssignmentExpert.com