Answer on Question #74242 – Math – Quantitative Methods

Question

For h=0.5,. 25,.125 solve integration from $\int_0^1 \frac{1}{1+x^2}dx$ and improve the accuracy by Romberg integration. Compare your value with the real value.

Solution

Evaluate an integral $\int_0^1 \frac{1}{1+x^2}dx$ and improve the accuracy by Romberg integration. We begin by using the Trapezoidal Rule, or, equivalently, the Composite Trapezoidal Rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f_0 + 2\sum_{i=1}^{N-1} f_i + f_N \right], h = \frac{b-a}{N}, \ x_i = x_0 + ih, \ \ x_0 = a, \ x_N = b$$

We get

N = 1 , h = 1, $I_T = \frac{h}{2}[f_0 + f_1] =$ 0.750000.

N = 2 , h = $\frac{1}{2}$, $I_T = \frac{h}{2}[f_0 + 2f_1 + f_2] = 0.775000$

N = 4 , h = $\frac{1}{4}$, $I_T = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + 2f_3 + f_4] = 0.782794$

N = 8 , h = $\frac{1}{8}$, $I_T = \frac{h}{2}[f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + 2f_6 + 2f_7 + f_8] = 0.784752$

N = 8 , h = $\frac{1}{8}$,

$$I_T = \frac{h}{2}\left[\begin{matrix} f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + 2f_5 + 2f_6 + 2f_7 + 2f_8 + 2f_9 + 2f_{10} \\ + 2f_{11} + 2f_{12} + 2f_{13} + 2f_{14} + 2f_{15} + f_{16} \end{matrix} \right] = 0.785452$$

Exact solution of the given integral is as follows:

$$\int_0^1 \frac{1}{1+x^2}dx = \left[\tan^{-1} x\right]_0^1 = \tan^{-1} 1 - \tan^{-1} 0 = 0.785398 \text{ in radians.}$$

Compare. Approximate value is 0.785452 when h=1/16 and the real value is 0.785398.

Using trapezoidal rule with Romberg integration to achieve the accuracy of 10^{-6}

$$R_1 = \frac{h^2}{12} f''(\xi), \quad 0 < \xi < 1$$

Since $f(x) = \frac{1}{1+x^2}$ *for* $0 \le x \le 1$ For achieving accuracy of 10^{-6}, we require h=0.007. hence at least (1-0)/0.007=144 function evaluations to achieve this accuracy if trapezoidal rule is used directly.