Question #74146

Derive a suitable numerical differentiation formula of 0(h2) to find f''(2. 4) with h =0.1 given the table f(0.1)=3.41, f(1.2) =2.68, f(2.4)=1. 37, f(3.9) =-1. 48.

Answer:

Using Taylor Expansion technique:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) \ldots,$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) \ldots.$$

Thus

$$f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \frac{h^4}{12}f^{(4)}(x) + \cdots,$$

$$f(x+h) + f(x-h) - 2f(x) = h^2 f''(x) + \frac{h^4}{12}f^{(4)}(x) + \cdots,$$

$$\frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x) + \frac{h^2}{12}f^{(4)}(x) + \cdots,$$

$$\frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''(x) + O(h^2).$$

So

$$f''(x) \approx \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}$$

with truncation error of order $O(h^2)$.

For x = 2.4 and h = 0.1

$$f(2.4) = 1.37,$$

$$f(2.4+0.1) = f(2.5) = 1.37 - (1.37 + 1.48)\frac{2.5 - 2.4}{3.9 - 2.4} = 1.18,$$

$$f(2.4-0.1) = f(2.3) = 1.37 + (2.68 - 1.37)\frac{2.4 - 2.3}{2.4 - 1.2} = 1.48,$$

$$f''(x) = \frac{1.18 + 1.48 - 2 \cdot 1.37}{0.1^2} = -8.$$