Question #74129 - Math - Quantitative Methods

using the classical R-K method of O(h4) calculate approximate solution of the IVP y'=1-x+4y, y(0) =1 at x=0.6, taking h=0.1 and 0.2. use extrapolation technique to improve the accuracy.

Solution: initial value problem

$$y' = f(x, y) = 1 - x + 4y, \qquad y(x_0) = y_0 = 1, \qquad x_0 = 0.6$$

In general a Runge–Kutta method of order 4 can be written as:

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = f(x_n, y_n) = 1 - x_n + 4y_n$$

$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right) = 1 - x_n - \frac{h}{2} + 4\left(y_n + \frac{h}{2}(1 - x_n + 4y_n)\right)$$
$$= 1 + \frac{3}{2}h - x_n(1 + 2h) + 4y_n(1 + 2h) = 1 + \frac{3}{2}h + (1 + 2h)(4y_n - x_n)$$

$$k_3 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right) = 1 - x_n - \frac{h}{2} + 4\left(y_n + \frac{h}{2}\left(1 + \frac{3}{2}h + (1 + 2h)(4y_n - x_n)\right)\right)$$
$$= 1 + \frac{3}{2}h + 3h^2 + (4y_n - x_n)(1 + 2h + 4h^2)$$

$$k_4 = f(x_n + h, y_n + hk_3)$$
$$= 1 - x_n - h + 4\left(y_n + h\left(1 + \frac{3}{2}h + 3h^2 + (4y_n - x_n)(1 + 2h + 4h^2)\right)\right)$$
$$= 1 + 3h + 6h^2 + 12h^3 + (4y_n - x_n)(1 + 4h + 8h^2 + 16h^3)$$

$$k_1 + 2k_2 + 2k_3 + k_4$$
$$= 1 + 2\left(1 + \frac{3}{2}h\right) + 2\left(1 + \frac{3}{2}h + 3h^2\right) + 1 + 3h + 6h^2 + 12h^3$$
$$+ (4y_n - x_n)(1 + 2(1 + 2h) + 2(1 + 2h + 4h^2) + (1 + 4h + 8h^2$$
$$+ 16h^3)) = 6 + 9h + 12h^2 + 12h^3 + (4y_n - x_n)(6 + 12h + 16h^2 + 16h^3)$$
$$= (4y_n - x_n + 1)(6 + 9h + 12h^2 + 12h^3) + (4y_n - x_n)(3h + 4h^2 + 4h^3)$$

Let

$$g(h, x_n, y_n) = h(k_1 + 2k_2 + 2k_3 + k_4)$$

For step h1=0.1

$$g(0.1, x_n, y_n) = 0.7032 - 0.7376x_n + 2.9504y_n$$

$$y_{n+1}(0.1) = y_n(0.1) + g(0.1, x_n, y_n)/6$$

For step h2=0.2

$$g(0.2, x_n, y_n) = 1.6752 - 1.8336x_n + 7.3344y_n$$

$$y_{n+1}(0.2) = y_n(0.2) + g(0.2, x_n, y_n)/6$$

To calculate the error of the solution, we use the analytic solution of IVP (y(x)-exact value):

$$y(x) = \frac{4x - 3 + 19 * e^{4x}}{16}$$

$$err_{n+1}(0.1) = y(x_{n+1}) - y_{n+1}(0.1)$$

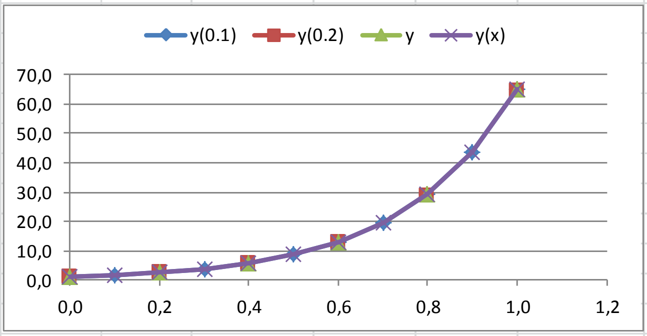
$$err_{n+1}(0.2) = y(x_{n+1}) - y_{n+1}(0.2)$$

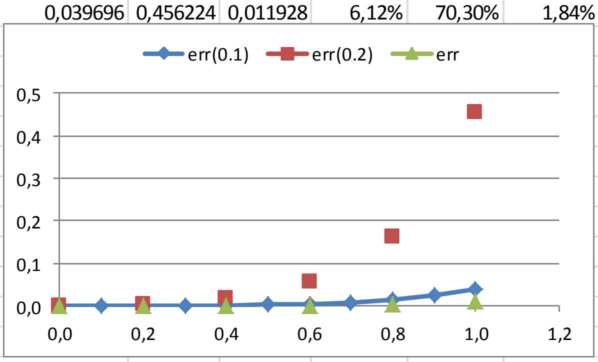
$$err_{n+1} = y(x_{n+1}) - y_{n+1}$$

Where y_{n+1} approximate solution of the IVP with use extrapolation technique on the step n+1. Richardson extrapolation technique (h1=h2/2):

$$y_{n+1} = y_{n+1}(0.1) + \frac{y_{n+1}(0.1) - y_{(n+1)*2}(0.2)}{2^4 - 1}$$

n	x	y(0.1)	y(0.2)	y	y(x)	err(0.1)	err(0.2)	err	err(0.1) %	err(0.2) %	err %
0,000000	0,000000	1,000000	1,000000	1,000000	1	0,000000	0,000000	0,000000	0,00%	0,00%	0,00%
1,000000	0,100000	1,608933			1,609041828	0,000108					
2,000000	0,200000	2,505006	2,501600	2,505233	2,505329853	0,000324	0,003730	0,000097	1,29%	14,89%	0,39%
3,000000	0,300000	3,829415			3,830138846	0,000724					
4,000000	0,400000	5,792785	5,777636	5,793795	5,794226004	0,001441	0,016590	0,000431	2,49%	28,63%	0,74%
5,000000	0,500000	8,709318			8,712004117	0,002687					
6,000000	0,600000	13,047713	12,997178	13,051082	13,05252195	0,004809	0,055344	0,001440	3,68%	42,40%	1,10%
7,000000	0,700000	19,507148			19,51551804	0,008370					
8,000000	0,800000	29,130609	28,980768	29,140599	29,14487961	0,014270	0,164111	0,004281	4,90%	56,31%	1,47%
9,000000	0,900000	43,473954			43,4979034	0,023949					
10,000000	1,000000	64,858107	64,441579	64,885875	64,89780316	0,039696	0,456224	0,011928	6,12%	70,30%	1,84%





Answer provided by https://www.AssignmentExpert.com