

Answer on Question #74061 – Math – Differential Equations

Question

$$\frac{dy}{dx} = \frac{y-x}{x-4y}$$

Solution

Transforming the right side we get:

$$\frac{y-x}{x-4y} = \frac{\frac{y}{x} - 1}{1 - 4\frac{y}{x}}$$

Now let  $\frac{y}{x} = t$ , then  $y = tx$ , and  $\frac{dy}{dx} = \frac{dt}{dx}x + t$ . So we have:

$$\frac{dt}{dx}x + t = \frac{t-1}{1-4t}$$

$$\frac{dt}{dx}x = \frac{t-1}{1-4t} - t = \frac{t-1}{1-4t} - \frac{t(1-4t)}{1-4t} = \frac{t-1-t+4t^2}{1-4t} = \frac{4t^2-1}{1-4t}$$

$$\frac{dx}{x} = \frac{1-4t}{4t^2-1} dt = \frac{dt}{(2t)^2-1^2} - \frac{t}{t^2-\frac{1}{4}} dt$$

$$\text{Integrating: } \int \frac{dx}{x} = \int \frac{dt}{(2t)^2-1^2} - \int \frac{t dt}{t^2-\frac{1}{4}}$$

$$\text{We've got: } \ln|x| = \frac{1}{2} \ln \left| \frac{2t-1}{2t+1} \right| - \frac{1}{2} \ln \left| t^2 - \frac{1}{4} \right| + \ln|C|$$

$$\text{Thus: } x = C \sqrt{\frac{2t-1}{(2t+1)(t^2-\frac{1}{4})}} = C \sqrt{\frac{t-\frac{1}{2}}{(t+\frac{1}{2})(t-\frac{1}{2})(t+\frac{1}{2})}} = C \frac{1}{t+\frac{1}{2}}$$

Remembering that  $t = \frac{y}{x}$ , we have:  $\frac{y}{x} + \frac{1}{2} = \frac{C}{x}$ , and  $y = C - \frac{x}{2}$

$$\text{Answer: } y = C - \frac{x}{2}$$