Answer to Question #74060, Math / Differential Equations

Find the complete integral of the equation

$$\frac{\partial z}{\partial x_1}\cdot\frac{\partial z}{\partial x_2}\cdot\frac{\partial z}{\partial x_3}=z^3x_1x_2x_3$$

Solution.

$$\left(\frac{1}{z}\frac{\partial z}{\partial x_1}\right)\left(\frac{1}{z}\frac{\partial z}{\partial x_2}\right)\left(\frac{1}{z}\frac{\partial z}{\partial x_3}\right)=x_1x_2x_3$$

Let

$$\frac{1}{z}dz=dZ \quad\Rightarrow\quad \ln z=Z$$

Then:

$$\left(\frac{\partial Z}{\partial x_1}\right)\left(\frac{\partial Z}{\partial x_2}\right)\left(\frac{\partial Z}{\partial x_3}\right)=x_1x_2x_3$$

$$P_1P_2P_3=x_1x_2x_3$$

$$P_1P_2P_3-x_1x_2x_3=0$$

$$f(x_1,x_2,x_3,P_1,P_2,P_3)=P_1P_2P_3-x_1x_2x_3=0$$

Jacobi's equations are

$$\frac{dP_1}{\partial f/\partial x_1}=\frac{dx_1}{-\partial f/\partial P_1}=\frac{dP_2}{\partial f/\partial x_2}=\frac{dx_2}{-\partial f/\partial P_2}=\frac{dP_3}{\partial f/\partial x_3}=\frac{dx_3}{-\partial f/\partial P_3}$$

$$\frac{dP_1}{-x_2x_3}=\frac{dx_1}{-P_2P_3}=\frac{dP_2}{-x_1x_3}=\frac{dx_2}{-P_1P_3}=\frac{dP_3}{-x_1x_2}=\frac{dx_3}{-P_1P_2}$$

Since

$$P_2P_3=\frac{x_1x_2x_3}{P_1}$$

Then:

$$\frac{dP_1}{-x_2x_3}=\frac{dx_1}{-(x_1x_2x_3/P_1)}$$

Answer to Question #74060, Math / Differential Equations

$$\frac{dP_1}{P_1} = \frac{dx_1}{x_1} \Rightarrow \ln P_1 = \ln x_1 + \ln a_1$$

Thus:

$$P_1 = a_1 x_1$$

$$P_2 = a_2 x_2$$

$$P_3 = x_3 / a_1 a_2$$

$$dZ = P_1 dx_1 + P_2 dx_2 + P_3 dx_3$$

$$dZ = a_1 x_1 dx_1 + a_2 x_2 dx_2 + \frac{x_3}{a_1 a_2} dx_3$$

$$Z = \ln z = \frac{a_1 x_1^2}{2} + \frac{a_2 x_2^2}{2} + \frac{x_3^2}{2 a_1 a_2} + \frac{a_3}{2}$$

Answer:

$$2 \ln z = a_1 x_1^2 + a_2 x_2^2 + \frac{x_3^2}{a_1 a_2} + a_3$$