Answer on Question #74057 – Math – Differential Equations

Question

1. A certain population is known to be growing at a rate given by the logistic equation $\frac{dx}{dt} = x(b - ax)$. Show that the minimum rate of growth will occur when the population is equal to half the equilibrium size, that is when the population is $\frac{b}{2a}$.

Solution

Let's find the minimum rate of growth, so the function $x(b - ax)$ should be minimized. The derivative of the rate must be zero:

$$\frac{d(bx - ax^2)}{dx} = 0$$

$$b - 2ax = 0$$

$$x = \frac{b}{2a}$$

$\frac{d^2(bx - ax^2)}{dx^2} = -2a < 0$ if $a > 0$, hence in this case we get a maximum;

$\frac{d^2(bx - ax^2)}{dx^2} = -2a > 0$ if $a < 0$, hence in this case we get a minimum.

Such a population x is equal to half the equilibrium size.

Answer: the minimum rate of growth will occur when population is $\frac{b}{2a}$.