Answer on Question #74056 – Math – Discrete Mathematics

Question

1. In how many ways can 30 identical balls be distributed into 7 distinct boxes (numbered box 1… box 7) subject to the following conditions?
(a) With no constraints.

Solution

We have to write 30 numbers $x_1, x_2, x_3, x_4, x_5, x_6, x_7,$ that correspond to the numder of balls that will be placed in boxes $1, 2, 3, 4, 5, 6,$ and 7 respectively. Since, they should add up to 30, we want to determine the number of solutions to the equation:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 30$$

subject to $x_i \geq 0$ (practically that means no constraints).

We suppose that empty boxes are allowed. Find the number of distributions of n objects among r boxes, if empty boxes are allowed.

Let's model our problem using the following imaginary picture.

Assume, objects are dots on a line (so, we have n dots), and vertical separators that are positioned between these dots signify the boundaries between the boxes (so, we have $r-1$ separators to model r boxes).

Any position of these $n+r-1$ elements would model some distribution of objects in boxes. So, we have $(n+r-1)!$ permutations.

The objects are identical. That means that any permutation of dots that leaves the mutual configuration of dots and separators unchanged produces the same distribution of objects among boxes. The same distribution we have counted $n!$ times. So, we have to reduce our number of permutations by a factor of $n!$

That results in $\dfrac{(n+r-1)!}{n!}$ distributions.

The separators also can be permuted without distorting the distribution of objects among boxes. That means that another division must be used to compensate for over-counting. There are $(r-1)!$ such permutations of $r-1$separators that result in the same object distribution among boxes.

That leaves us with the number of distributions of

$$\frac{(n+r-1)!}{n!\,(r-1)!}$$

Therefore, the assignments of 30 identical objects to 7 boxes is equivalent to the number of non-negative integer solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 30$$

which is

$$\binom{30+7-1}{7-1} = \frac{(36)!}{30!\,6!} = \frac{36(35)(34)(33)(32)(31)}{1(2)(3)(4)(5)(6)} = 1947792$$

Answer: 1947792 ways.