**Answer on Question #74012 – Math – Quantitative Methods**

**Question**

find the dominant eigenvalue and the corresponding eigenvector for the matrix $A = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ using five iterations of the power method and taking y(0)= $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ as the initial vector.

**Solution**

We begin with an initial non-zero approximation of dominant eigenvector $x_0$ as $x_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and we obtain the following approximations as

$$x_1 = A\tilde{x}_1 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} = 10 \begin{pmatrix} 1 \\ 0.8 \\ 0.1 \end{pmatrix}$$

We take out the largest element of the resultant matrix and will be our new initial vector. Proceeding in this manner we obtain a series of approximations as follows:

$$x_2 = A\tilde{x}_2 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.8 \\ 0.1 \end{pmatrix} = \begin{pmatrix} 7.2 \\ 5.4 \\ -0.8 \end{pmatrix} = 7.2 \begin{pmatrix} 1 \\ 0.75 \\ -0.11 \end{pmatrix}$$

$$x_3 = A\tilde{x}_3 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.75 \\ -0.11 \end{pmatrix} = \begin{pmatrix} 6.5 \\ 4.75 \\ -1.22 \end{pmatrix} = 6.5 \begin{pmatrix} 1 \\ 0.730 \\ -0.187 \end{pmatrix}$$

$$x_4 = A\tilde{x}_4 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.730 \\ -0.187 \end{pmatrix} = \begin{pmatrix} 6.22 \\ 4.49 \\ -1.37 \end{pmatrix} = 6.22 \begin{pmatrix} 1 \\ 0.7218 \\ -0.2202 \end{pmatrix}$$

$$x_5 = A\tilde{x}_5 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.7218 \\ -0.2202 \end{pmatrix} = \begin{pmatrix} 6.1052 \\ 4.3834 \\ -1.4404 \end{pmatrix} = 6.1052 \begin{pmatrix} 1 \\ 0.7179 \\ -0.2359 \end{pmatrix}$$

$$x_6 = A\tilde{x}_6 = \begin{pmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0.7179 \\ -0.2359 \end{pmatrix} = \begin{pmatrix} 6.05 \\ 4.33 \\ -1.47 \end{pmatrix} = 6.05 \begin{pmatrix} 1 \\ 0.715 \\ -0.242 \end{pmatrix}$$

Therefore, the dominant eigen-value is approximately **6.05** and the corresponding eigenvector is

$$\begin{pmatrix} 1 \\ 0.715 \\ -0.242 \end{pmatrix}.$$