Answer on Question #73969 – Math – Real Analysis

Question

Using delta - epsilon definition prove that lim(x-->0) (sinx/x) = 1

Solution

$For\ x > 0:\ x - \frac{x^3}{3!} < sinx < x.$

$For\ x < 0:\ x < sinx < x - \frac{x^3}{3!}.$

$For\ x = 0:\ sinx = x.$

Thus, $|sinx - x| \leq \frac{x^3}{3!}$ or $\left|\frac{sinx}{x} - 1\right| \leq \frac{x^2}{6}.$

Then, whenever $|x - 0| = |x| < \delta,$ $\left|\frac{sinx}{x} - 1\right| \leq \frac{x^2}{6} < \frac{\delta^2}{6}.$

So, for every $\varepsilon > 0$, no matter how small, we can choose $\delta = \sqrt{6\varepsilon}$, such that

$|x - 0| < \delta$ implies $\left|\frac{sinx}{x} - 1\right| < \varepsilon$ we have proved that $\lim_{x \to 0} \frac{sinx}{x} = 1.$