Answer on Question #73856 – Math – Linear Algebra

Question

Find the inverse of the matrix $A=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & 4\\ 1 & 2 & 2\end{bmatrix}$ by Gauss - Jordan method.

Solution

Take two matrices: A and the identity $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$. Reduce the matrix A to the identity matrix by the Gauss-Jordan method. After applying each operation to the first matrix, we apply the same operation to the second one. When the reduction of the first matrix to a single form is completed, the second matrix will be equal to A^{-1}.

Step 1. Subtract the first line from the second.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & -1 & 3\\ 1 & 2 & 2\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Step 2. Subtract the first line from the third.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & -1 & 3\\ 0 & 3 & 1\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}.$$

Step 3. Multiply the second line by -1 and third line by $\frac{1}{3}$.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -3\\ 0 & 1 & \frac{1}{3}\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & 0\\ -\frac{1}{3} & 0 & \frac{1}{3}\end{bmatrix}.$$

Step 4. Subtract the second line from the third.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -3\\ 0 & 0 & 3\frac{1}{3}\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & 0\\ -\frac{4}{3} & 1 & \frac{1}{3}\end{bmatrix}.$$

Step 5. Multiply the third line by $\frac{3}{10}$.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & -3\\ 0 & 0 & 1\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & 0\\ -\frac{4}{10} & \frac{3}{10} & \frac{1}{10}\end{bmatrix}.$$

Step 6. Subtract the third line, multiplied by -3, from the second.

$$A=\begin{bmatrix}1 & -1 & 1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix};\ I=\begin{bmatrix}1 & 0 & 0\\ -\frac{2}{10} & -\frac{1}{10} & \frac{3}{10}\\ -\frac{4}{10} & \frac{3}{10} & \frac{1}{10}\end{bmatrix}.$$

Step 7. Subtract the second line, multiplied by -1 and the third line from the first.

$$A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix};\ I=\begin{bmatrix}\frac{12}{10} & -\frac{4}{10} & \frac{2}{10}\\ -\frac{2}{10} & -\frac{1}{10} & \frac{3}{10}\\ -\frac{4}{10} & \frac{3}{10} & \frac{1}{10}\end{bmatrix}=A^{-1}.$$

Answer: $A^{-1}=\begin{bmatrix}1.2 & -0.4 & 0.2\\ -0.2 & -0.1 & 0.3\\ -0.4 & 0.3 & 0.1\end{bmatrix}.$