Answer on Question #73853 – Math – Quantitative Methods

Question

Estimate the eigenvalues of the matrix

$$\begin{pmatrix} 1 & -2 & 3 \\ 6 & -13 & 18 \\ 4 & -10 & 14 \end{pmatrix}$$

using the Gershgorin bounds. Draw a rough sketch of the region where the eigenvalues lie.

Solution

Gershgorin circle theorem: Every eigenvalue of a square matrix lies in at least one of the Gershgorin discs C_i. The possible range of the eigenvalues is defined by the outer borders of the union of all discs

$$C = \bigcup_{i=1}^{n} C_i$$

where

$$C_i = \{|c - a_{ii}| \leq r_i\}$$

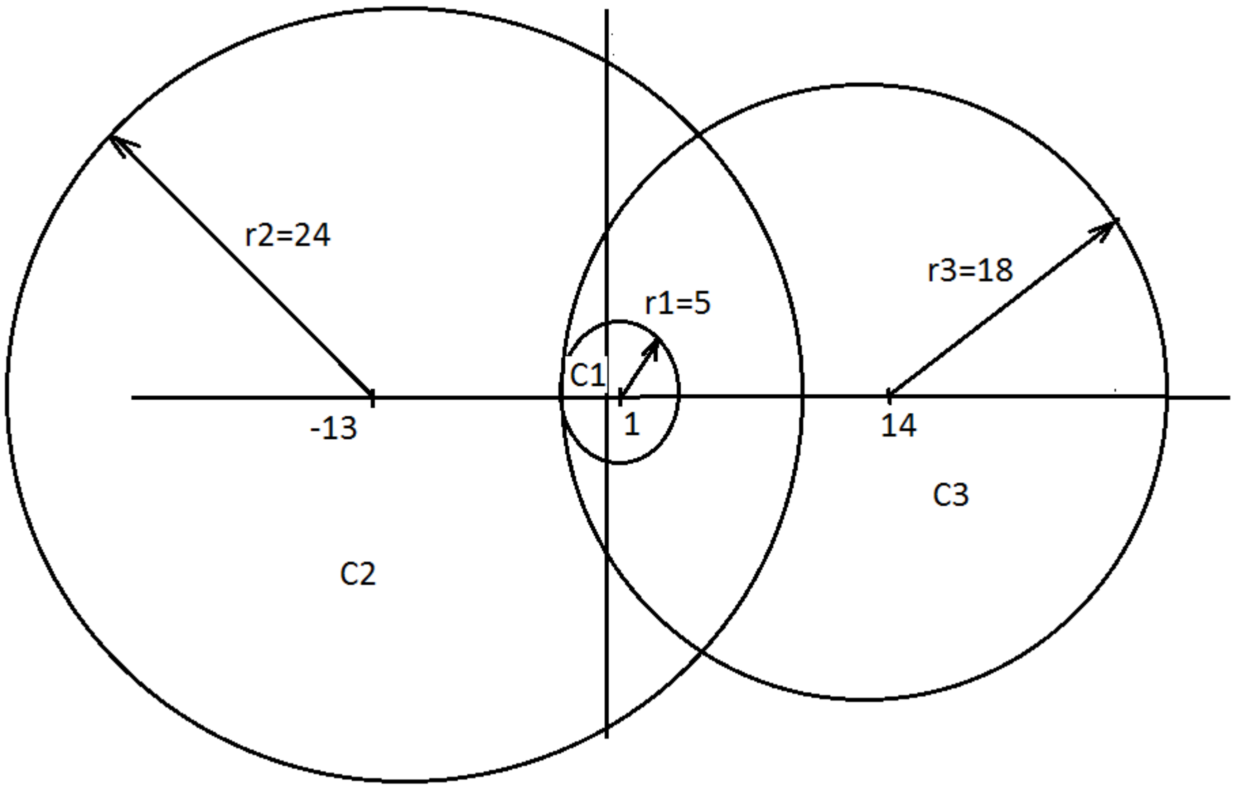
$$r_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|$$

There are three Gershgorin discs in this matrix:

C_1 with the centre point $a_{11} = 1$ and radius $r_1 = 2 + 3 = 5$

C_2 with the centre point $a_{22} = -13$ and radius $r_2 = 6 + 18 = 24$

C_3 with the centre point $a_{33} = 14$ and radius $r_3 = 4 + 14 = 18$



Answer: region where the eigenvalues lie:

$$C_1 \cup C_2 \cup C_3$$