

Answer on Question #73846 – Math – Calculus

Question

Write the equation for the ellipse with foci at (5,-5) and (1,-5) and major axis of length 14?

Solution

The equation of the ellipse is

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

a, b – semi-major and semi-minor axes. a=14/2=7

x0, y0 – center of the ellipse

$$b = a \cdot \sqrt{1 - \frac{c^2}{a^2}} = \sqrt{a^2 - c^2}$$

c – focal distance (F1=(5,-5)=(x1,y1) and F2=(1,-5)=(x2,y2))

$$c = \frac{|F_1F_2|}{2} = \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{2} = \frac{\sqrt{16 + 0}}{2} = 2$$

Then $b = \sqrt{49 - 4} = \sqrt{45}$

$$x_0 = \frac{x_1 + x_2}{2} = \frac{6}{2} = 3$$

$$y_0 = \frac{y_1 + y_2}{2} = \frac{-10}{2} = -5$$

and equation of the ellipse is

$$\frac{(x - 3)^2}{49} + \frac{(y + 5)^2}{45} = 1$$

Answer: $\frac{(x-3)^2}{49} + \frac{(y+5)^2}{45} = 1$