

## Answer on Question #73823 – Math – Calculus

### Question

Show that  $\Delta(\ln r) = \text{vector } r/r$

### Solution

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

$$\Delta(\ln(r)) = \nabla \cdot \nabla(\ln(r))$$

$$\frac{\partial}{\partial x}(\ln(r)) = \frac{1}{r} \frac{dr}{dx} = \frac{1}{r} \cdot \frac{1}{2\sqrt{x^2 + y^2 + z^2}} \cdot 2x = \frac{x}{r^2}$$

$$\frac{\partial}{\partial y}(\ln(r)) = \frac{y}{r^2}$$

$$\frac{\partial}{\partial z}(\ln(r)) = \frac{z}{r^2}$$

$$\nabla(\ln(r)) = \frac{\partial(\ln(r))}{\partial x}\vec{i} + \frac{\partial(\ln(r))}{\partial y}\vec{j} + \frac{\partial(\ln(r))}{\partial z}\vec{k} = \frac{\vec{r}}{r^2}$$

$$\begin{aligned}\Delta(\ln(r)) &= \nabla \cdot \nabla(\ln(r)) = \nabla \cdot \left( \frac{\vec{r}}{r^2} \right) = \frac{\partial}{\partial x} \left( \frac{x}{r^2} \right) + \frac{\partial}{\partial y} \left( \frac{y}{r^2} \right) + \frac{\partial}{\partial z} \left( \frac{z}{r^2} \right) \\ &= \left\{ \frac{\partial}{\partial x} \left( \frac{x}{r^2} \right) = \frac{1}{r^2} - \frac{2x^2}{r^4} \right\} = \frac{1}{r^2} - \frac{2x^2}{r^4} + \frac{1}{r^2} - \frac{2y^2}{r^4} + \frac{1}{r^2} - \frac{2z^2}{r^4} \\ &= \frac{3}{r^2} - \frac{2}{r^4} (x^2 + y^2 + z^2) = \frac{1}{r^2}\end{aligned}$$

**Answer:**  $\Delta(\ln(r)) = \frac{1}{r^2}$