Answer on Question #73821 – Math – Calculus

QUESTION

Show that for two scalar fields f and g:

$$\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right) + \vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right) = \vec{0}$$

SOLUTION

We recall some notation and formulas from the vector calculus.

1) For any vector $\vec{F}$

$$\vec{\nabla} \times \vec{F} \equiv curl(\vec{F})$$

2) For any scalar field φ and any vector $\vec{A}$

$$\vec{\nabla} \times (\varphi\vec{A}) = \varphi \cdot curl(\vec{A}) + (\vec{\nabla}(\varphi) \times \vec{A})$$

3) For any scalar field φ

$$curl\left(\vec{\nabla}(\varphi)\right) \equiv \vec{0}$$

(More information: https://en.wikipedia.org/wiki/Vector_calculus_identities)

4) For two vectors $\vec{A}$ and $\vec{B}$

$$(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A}) \leftrightarrow (\vec{A} \times \vec{B}) + (\vec{B} \times \vec{A}) = \vec{0}$$

(More information: https://en.wikipedia.org/wiki/Cross_product)

We transform expression

$$\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right) + \vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right)$$

1) $\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right)$

$$\vec{\nabla} \times \left(\underbrace{f}_{\varphi} \cdot \underbrace{\vec{\nabla}(g)}_{\vec{A}}\right) = (2\ formula) = f \cdot \underbrace{curl\left(\vec{\nabla}(g)\right)}_{=\vec{0}(3\ formula)} + \left(\vec{\nabla}(f) \times \vec{\nabla}(g)\right) =$$

$$= f \cdot \vec{0} + \left(\vec{\nabla}(f) \times \vec{\nabla}(g)\right) = \left(\vec{\nabla}(f) \times \vec{\nabla}(g)\right)$$

$$\boxed{\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right) = \left(\vec{\nabla}(f) \times \vec{\nabla}(g)\right)}$$

2) $\vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right)$

$$\vec{\nabla} \times \left(\underbrace{g}_{\varphi} \cdot \underbrace{\vec{\nabla}(f)}_{\vec{A}}\right) = (2\ formula) = g \cdot \underbrace{curl\left(\vec{\nabla}(f)\right)}_{=\vec{0}(3\ formula)} + \left(\vec{\nabla}(g) \times \vec{\nabla}(f)\right) =$$

$$= g \cdot \vec{0} + \left(\vec{\nabla}(g) \times \vec{\nabla}(f)\right) = \left(\vec{\nabla}(g) \times \vec{\nabla}(f)\right)$$

$$\boxed{\vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right) = \left(\vec{\nabla}(g) \times \vec{\nabla}(f)\right)}$$

Then,

$$\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right) + \vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right) = \left(\underbrace{\vec{\nabla}(f)}_{\vec{A}} \times \underbrace{\vec{\nabla}(g)}_{\vec{B}}\right) + \left(\underbrace{\vec{\nabla}(g)}_{\vec{B}} \times \underbrace{\vec{\nabla}(f)}_{\vec{A}}\right) = \vec{0}$$

Conclusion,

$$\boxed{\vec{\nabla} \times \left(f \cdot \vec{\nabla}(g)\right) + \vec{\nabla} \times \left(g \cdot \vec{\nabla}(f)\right) = \vec{0}}$$