Question

Determine whether the following vector field is solenoidal, irrotational or both

F= x^2.yi + xyzj - x^2.y^2.k

Solution

Since

$$div\ F\ =\ (\nabla \cdot F)\ =\ \partial_x(x^2 y)\ +\ \partial_y(xyz) + \partial_z(x^2 y^2) = 2xy + xz\ \neq 0\ \text{for all}$$

x,y,z

field F is not strictly solenoidal

And since

$$rot\ F\ =\ [\nabla \times F]\ =\ (2x^2 y - xy, 2xy^2, yx - x^2) \neq 0\ \text{for all x, y, z}$$

field F is not irrotational

That means that field F contains both irrotational and solenoidal part

Answer: both.