

Answer on Question #73772 – Math – Differential Equations

Question

Solve the differential Equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 3x^2$$

Solution

$$y'' + 3y' - 10y = 3x^2 \quad [1]$$

is a linear nonhomogeneous differential equation

$$y'' + 3y' - 10y = 0 \quad [2]$$

is a linear homogeneous differential equation

$$k^2 + 3k - 10 = 0 \quad [3]$$

$$k_1 = -5, \quad k_2 = 2$$

$$y^* = C_1 e^{-5x} + C_2 e^{2x} \quad - \text{ general solution of a linear homogeneous differential equation}$$

We consider the right-hand side of [1] :

$$f(x) = 3x^2$$

$\alpha = 0, \quad \beta = 0, \quad \Delta = \alpha \pm \beta i = 0$ does not coincide with solutions [3] of equation, so we seek the particular solution of the nonhomogeneous differential equation in the form

$$\bar{y} = Ax^2 + Bx + C$$

$$\bar{y}' = 2Ax + B$$

$$\bar{y}'' = 2A$$

$$2A + 6Ax + 3B - 10Ax^2 - 10Bx - 10C = 3x^2$$

$$-10Ax^2 + (6A - 10B)x + (2A + 3B - 10C) = 3x^2$$

$$\begin{cases} -10A = 3 \\ 6A - 10B = 0 \\ 2A + 3B - 10C = 0 \end{cases} \Rightarrow \begin{cases} A = -\frac{3}{10} \\ B = -\frac{9}{50} \\ C = -\frac{57}{500} \end{cases}$$

$$\bar{y} = -\frac{3}{10}x^2 - \frac{9}{50}x - \frac{57}{500}$$

the general solution of equation [1]:

$$y = y^* + \bar{y}$$

$$y = C_1 e^{-5x} + C_2 e^{2x} - \frac{3}{10}x^2 - \frac{9}{50}x - \frac{57}{500}$$

Answer: $y = C_1 e^{-5x} + C_2 e^{2x} - \frac{3}{10}x^2 - \frac{9}{50}x - \frac{57}{500}$