ANSWER ON QUESTION #73762 – MATH – CALCULUS

QUESTION

A jogger runs from her home to a point A, which is 6 km away. For there 6 km, she begins by running at a constant speed till she reaches a hilly portion 2 km from her home. Here her speed slows down while she runs up the hill, which is a 1-km run. Then she speeds up while running down the hill. The last 2 km of the run are again at constant speed. Draw a graph to show the jogger's speed as a function of the distance from her home. Also find the range of this function.

SOLUTION

There are two different cases of how to change the speed of the jogger.

1 case:

The graph of the jogger's speed is shown on the Figure 1.

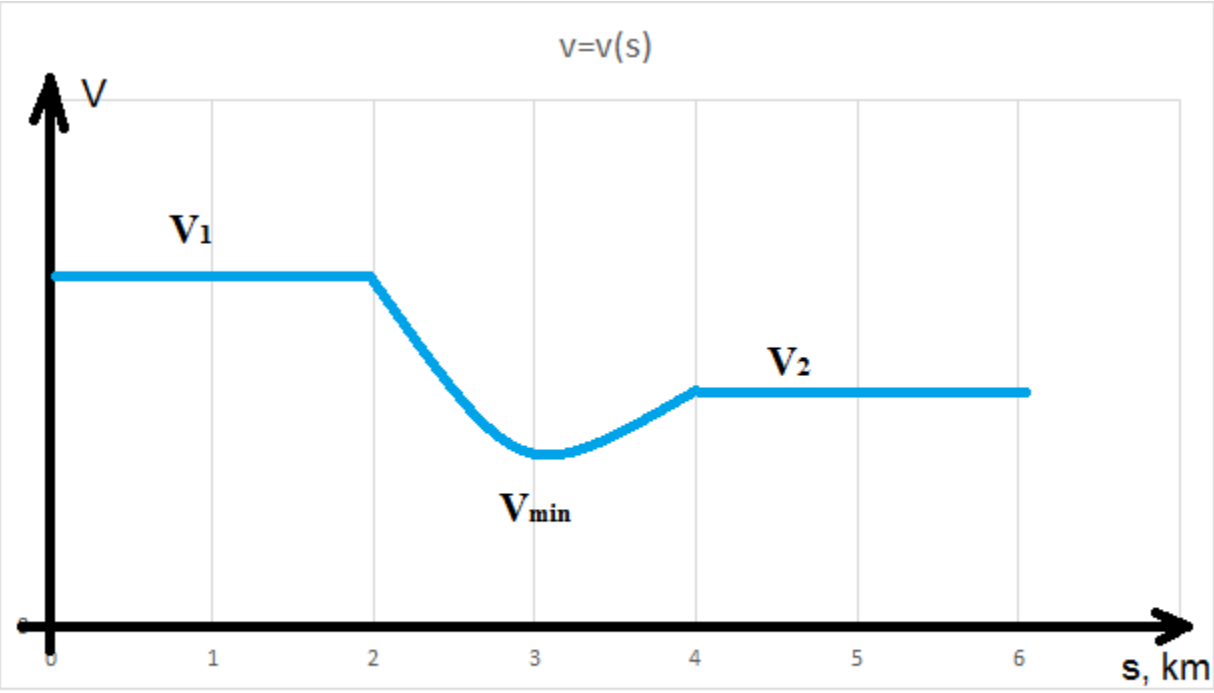

Figure 1

As we can see the initial jogger's speed V_1 is constant for the first $2\ km$ of the distance. The next $1\ km$ the speed decreases to its minimal value V_{min}. Then the speed increases up to V_2 and stay constant for the last $2\ km$ of distance. Suppose $V_2 < V_1$.

The Figure 1 shown that the values of the jogger's speed function are between V_{min} and V_1. So, the range of this function is $[V_{min}, V_1]$.

$$\boxed{V_{min} \leq V(s) \leq V_1}$$

2 case:

The graph of the jogger's speed is shown on the Figure 2.

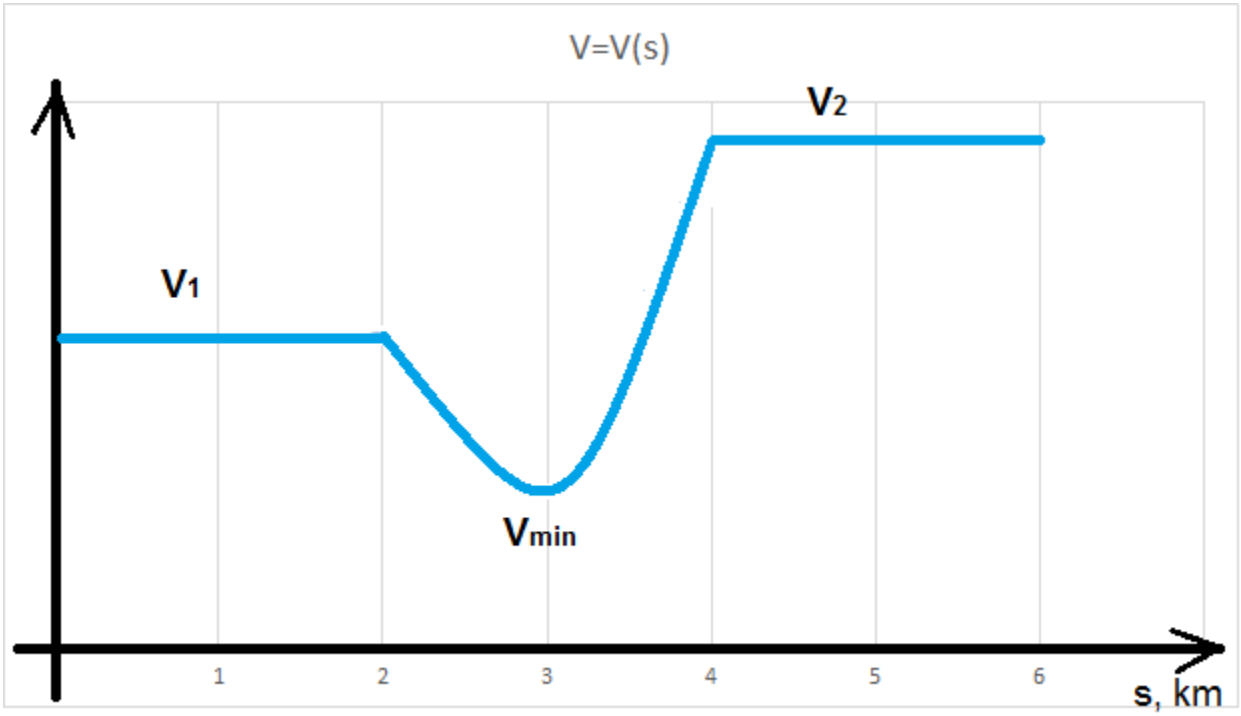

Figure 2

As we can see the initial jogger's speed V_1 is constant for the first $2\ km$ of the distance. The next $1\ km$ the speed decreases to its minimal value V_{min}. Then the speed increases up to V_2 and stay constant for the last $2\ km$ of distance. Suppose $V_1 < V_2$.

The Figure 2 shown that the values of the jogger's speed function are between V_{min} and V_2. So, the range of this function is $[V_{min}, V_2]$.

$$\boxed{V_{min} \leq V(s) \leq V_2}$$

Conclusion.

These two cases can be described by a single formula:

The range of the jogger's speed function is $[V_{min}, \max(V_1, V_2)]$.

$$\boxed{V_{min} \leq V(s) \leq \max(V_1, V_2)}$$

ANSWER

$$V_{min} \leq V(s) \leq \max(V_1, V_2)$$