Answer on Question #73746 – Math – Calculus

Question

The original function used to model the cost of producing x PortaBoys Game Systems was

$$C(x) = 80x + 150.$$

While developing their newest game, Sasquatch Attack!, the makers of the PortaBoy revised their cost function using a cubic polynomial. The new cost of producing x PortaBoys is given by

$$C(x) = .03x^3 - 4.5x^2 + 221x + 250.$$

Market research indicates that the demand function

$$p(x) = -1.5x + 250$$

remains unchanged. Find the production level x that maximizes the profit made by producing and selling x PortaBoys. (Round your answer to the nearest whole number.)

Solution

Total cost of producing:

$$TC(x) = 0.03x^3 - 4.5x^2 + 221x + 250$$

where x =production level.

Demand function:

$$p(x) = -1.5x + 250$$

where $p(x)$ = price, x =production level.

Total revenue is given by

$$TR(x) = p(x)x = -1.5x^2 + 250x$$

Profit

$$P(x) = TR(x) - TC(x)$$

$P(x) = -1.5x^2 + 250x - (0.03x^3 - 4.5x^2 + 221x + 250) =$

$= -0.03x^3 + 3x^2 + 29x - 250$

Find the marginal profit

$P'(x) = -0.09x^2 + 6x + 29$

Find the critical point(s)

$$P'(x) = 0 => -0.09x^2 + 6x + 29 = 0$$

$$x_1 = \frac{-6 - \sqrt{(6)^2 - 4(-0.09)(29)}}{2(-0.09)} = \frac{3 + \sqrt{11.61}}{0.09}$$

$$x_2 = \frac{-6 + \sqrt{(6)^2 - 4(-0.09)(29)}}{2(-0.09)} = \frac{3 - \sqrt{11.61}}{0.09}$$

$$If\ x < \frac{3 - \sqrt{11.61}}{0.09}\ , then\ P'(x) < 0, P(x)\ decreases$$

$$If\ \frac{3 - \sqrt{11.61}}{0.09} < x < \frac{3 + \sqrt{11.61}}{0.09}\ , then\ P'(x) > 0, P(x)\ increases$$

$$If\ x > \frac{3 + \sqrt{11.61}}{0.09}\ , then\ P'(x) < 0, P(x)\ decreases$$

The profit function has maximum at

$$x = \frac{3 + \sqrt{11.61}}{0.09}$$

$x = \frac{3 + \sqrt{11.61}}{0.09} \approx 71.2$

Find

$P(71) = -0.03(71)^3 + 3(71)^2 + 29(71) - 250 = 6194.67$

$P(72) = -0.03(72)^3 + 3(72)^2 + 29(72) - 250 = 6192.56$

Since $6192.56 < 6194.67$, we take $x = 71$.

Answer: 71.