Answer on Question #73691 – Math – Calculus

Question

Using Stokes' Theorem evaluate the line integral integrate at C F.dl where F= you+ xz^3 .j - zy^3 .k and C is circle x^2+y^2=5 in the plane z=-3.

Solution

$$\vec{F}(x,y,z) = \vec{\imath} + x \cdot z^3 \cdot \vec{\jmath} - z \cdot y^3 \cdot \vec{k}$$

Boundary (C):

$$x^2 + y^2 = 5 \; and \;\; z = -3$$

$$(\frac{x}{\sqrt{5}})^2 + (\frac{y}{\sqrt{5}})^2 = 1$$

Parameterize ($0 \le t \le 2\pi$):

$$x = \sqrt{5}\cos(t)$$

$$y = \sqrt{5}\sin(t)$$

$$\frac{dx}{dt} = -\sqrt{5}\sin(t)$$

$$\frac{dy}{dt} = \sqrt{5}cos(t)$$

A vector equation of C is:

$$\vec{r}(t) = \sqrt{5}\cos(t) \cdot \vec{\imath} + \sqrt{5}\sin(t) \cdot \vec{\jmath} - 3 \cdot \vec{k}$$

$$\vec{r}'(t) = -\sqrt{5}\sin(t) \cdot \vec{\imath} + \sqrt{5}\cos(t) \cdot \vec{\jmath}$$

Then

$$\vec{F}(r(t)) = \vec{\imath} + \sqrt{5}\cos(t) \cdot (-3)^3 \cdot \vec{\jmath} - (-3) \cdot (\sqrt{5}\sin(t))^3 \cdot \vec{k}$$

Thus, by Stokes Theorem:

$$\iint_S curl\, F \cdot dS = \oint_C F \cdot dr = \oint_0^{2\pi} F(r(t)) \cdot r'(t)dt$$

$$= \oint_0^{2\pi} -\sqrt{5}\sin(t)\,dt + \oint_0^{2\pi} (-27)\sqrt{5}\cos(t)\sqrt{5}\cos(t)\,dt$$

$$= -\sqrt{5}\oint_0^{2\pi} \sin(t)\,dt - 135\oint_0^{2\pi} cos^2(t)\,dt$$

$$= \left\{cos^2(t) = \frac{1}{2}\cos(2t) + \frac{1}{2} \quad let \quad u = 2t \quad then \quad du\right.$$

$$\left.= 2dt\ and\ \int \frac{1}{2}\cos(u)\,du = \frac{\sin(2t)}{2}\right\}$$

$$= \sqrt{5}\cos(t)\,|_0^{2\pi} - 135\frac{\sin(2t)}{4}|_0^{2\pi} - 135\frac{t}{2}|_0^{2\pi} = -135\pi$$

Answer: -135π.