Answer on Question #73688 – Math – Calculus

Question

Calculate the work done by a force F=2xi+3yj in moving a particle once counterclockwise along the ellipse x^2/4 + y^2/9 =1 .

Solution

The work (W) done by a force (F) in moving along the curve (C) :

$$W = \int_C F \cdot dr = \oint_C P(x,y)dx + Q(x,y)dy$$

$$P(x,y) = 2x$$

$$Q(x,y) = 3y$$

Boundary of given ellipse:

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

Parameterize the ellipse ($0 \leq t \leq 2\pi$):

$$x = 2\cos(t)$$

$$y = 3\sin(t)$$

$$\frac{dx}{dt} = -2\sin(t)$$

$$\frac{dy}{dt} = 3cos(t)$$

Then:

$$W = \oint_C 2x\,dx + 3y\,dy$$

$$= \int_0^{2\pi} 2 \cdot 2 \cdot \cos(t) \cdot (-2) \cdot \sin(t)dt + 3 \cdot 3 \cdot \sin(t) \cdot 3 \cdot cos(t)dt$$

$$W = 19\int_0^{2\pi} \cos(t)\cdot\sin(t)\ dt$$

$$= \left\{let\ u = \sin(t)\ then\ du = \cos(t)\ dt\ and\ \int u\ du = \frac{u^2}{2}\right\}$$

$$= \frac{19}{2}(\sin(t))^2|_0^{2\pi} = 0$$

Answer: The work done $W = 0$.