Answer on Question #73681 – Math – Vector Calculus

QUESTION

For a particle undergoing circular motion with an angular velocity $\vec{\omega}$ in a circle of radius r show that

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 \vec{r}$$

SOLUTION

To solve this problem, we need to recall some formulas and facts:

1) The formula for the vector triple product (from a linear algebra)

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}\left(\vec{a} \cdot \vec{b}\right),$$

where

$$\vec{a} \cdot \vec{c} = |\vec{a}| \cdot |\vec{c}| \cdot \cos(\varphi)\ is\ the\ scalar(dot)\ product\ of\ vectors$$

$$\vec{a} \cdot \vec{a} = |\vec{a}| \cdot |\vec{a}| \cdot \cos(0°) = a \cdot a = a^2$$

(More information: https://en.wikipedia.org/wiki/Triple_product)

(More information: https://en.wikipedia.org/wiki/Dot_product)

2) An angular velocity

$$\vec{\omega} \perp \vec{r} \rightarrow \vec{\omega} \cdot \vec{r} = |\vec{\omega}| \cdot |\vec{r}| \cdot \cos(90°) = |\vec{\omega}| \cdot |\vec{r}| \cdot 0 = 0$$

$$\boxed{\vec{\omega} \perp \vec{r} \rightarrow \vec{\omega} \cdot \vec{r} = 0}$$

(More information: https://en.wikipedia.org/wiki/Angular_velocity)

In our case,

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\omega}(\vec{\omega} \cdot \vec{r}) - \vec{r}(\vec{\omega} \cdot \vec{\omega}) = \vec{\omega} \cdot 0 - \vec{r} \cdot \omega^2 = -\omega^2 \vec{r}$$

Thus,

$$\boxed{\vec{\omega} \times (\vec{\omega} \times \vec{r}) = -\omega^2 \vec{r}}$$