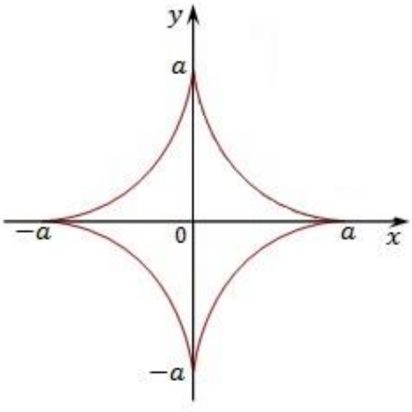
Answer on Question #73656 – Math – Calculus

Find the volume of the solid obtained by revolving the curve $x = acos^3\theta, y = asin^3\theta$ about the y-axis.

Solution



Let us consider the half of the astroid arranged symmetrically about the y-axis.

If $y = asin^3\theta = a\ then\ \theta = \frac{\pi}{2}$

If $y = asin^3\theta = 0\ then\ \theta = 0$

Use the formula

$$V = 2\pi \int_a^b x \cdot y(x)dx$$

$$dx = (-3acos^2\theta\ sin\theta)\ d\theta$$

$$V = -2\pi \int_{\frac{\pi}{2}}^0 acos^3\theta\ \cdot asin^3\theta \cdot\ (-3acos^2\theta\ sin\theta)\ d\theta = -6\pi a^3 \int_{\frac{\pi}{2}}^0 cos^5\theta\ \cdot sin^4\theta\ d\theta$$

$$\int_{\frac{\pi}{2}}^0 cos^5\theta\ \cdot sin^4\theta\ d\theta = \int_{\frac{\pi}{2}}^0 cos^4\theta\ \cdot sin^4\theta\ dsin\theta =$$

$$= \int_{\frac{\pi}{2}}^0 (1 - sin^2\theta)^2\ \cdot sin^4\theta\ dsin\theta = \frac{sin^5\theta}{5} - \frac{2sin^7\theta}{7} + \frac{sin^9\theta}{9}\Bigg|_{\frac{\pi}{2}}^0 = -\frac{8}{315}$$

$$-6\pi a^3 \int_{\frac{\pi}{2}}^0 cos^5\theta\ \cdot sin^4\theta\ d\theta = -6\pi a^3 \cdot \left(-\frac{8}{315}\right) = \frac{16}{105}\pi a^3$$

Then the volume of the whole body formed by the rotation of the astroid will be $V = \frac{32}{105}\pi a^3$

Answer: $V = \frac{32}{105}\pi a^3$.

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