# Answer on Question #73546 – Math – Differential Equations

## Question

show that the function

**1)** u(x,y)=tan^-1(y/x) is a solution of the two dimensional Laplace's equation.

**2)** u(x,t)=e^-(6t) cos2x is a solution of the one dimensional heat equation.

## Solution

**1)** $\frac{\partial u}{\partial x}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(-\frac{y}{x^2}\right)=-\frac{y}{x^2+y^2},\ \ \frac{\partial u}{\partial y}=\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x}=\frac{x}{x^2+y^2}.$

$$\frac{\partial^2 u}{\partial x^2}=-y\cdot\left(-\frac{1}{(x^2+y^2)^2}\right)\cdot 2x=\frac{2xy}{(x^2+y^2)^2},$$

$$\frac{\partial^2 u}{\partial y^2}=x\cdot\left(-\frac{1}{(x^2+y^2)^2}\right)\cdot 2y=-\frac{2xy}{(x^2+y^2)^2},$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{2xy}{(x^2+y^2)^2}-\frac{2xy}{(x^2+y^2)^2}=0.$$

**2)** $\frac{\partial u}{\partial x}=-2e^{-6t}sin2x,\ \ \frac{\partial u}{\partial t}=-6e^{-6t}cos2x.$

$$\frac{\partial^2 u}{\partial x^2}=\ -4e^{-6t}cos2x.$$

$$\frac{3}{2}\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}\ \ or\ \ \frac{\partial u}{\partial t}-k\frac{\partial^2 u}{\partial x^2}=0\ \ \ where\ k=\frac{3}{2}.$$