Answer on Question #73082 – Math – differential Equations

Question

(x+2)y''+xy'-y=0

Solution

(x+2)y''+xy'-y=0

Comparing the given equation with the form y''+P(x)y' + Q(x)y = 0

We get P(x)= $\frac{x}{x+2}$, Q(x)= $\frac{-1}{x+2}$

At x=0, both P(x) and Q(x) are analytic , hence at x=0 is an ordinary point.

Assume its solutions to be

$Y= a_0+a_1x+a_2x^2+a_3x^3+\dots\dots+a_nx^n+\dots\dots\dots$ (1)

Then $Y'= a_1+2a_2x+3a_3x^2+\dots\dots+na_nx^{n-1}+\dots\dots\dots$

$Y''= 2.1\ a_2+3.2.a_3x+\dots\dots+n(n-1)a_nx^{n-2}+\dots\dots\dots$

Substituting these values in the given differential equation, we get

$(x+2)(\ 2.1\ a_2+3.2.a_3x+\dots\dots+n(n-1)a_nx^{n-2}+\dots\dots\dots)+x(a_1+2a_2x+3a_3x^2+\dots\dots+na_nx^{n-1}+\dots\dots\dots)-\ a_0+a_1x+a_2x^2+a_3x^3+\dots\dots+a_nx^n+\dots\dots\dots=0$

Equating to zero, the various powers of x as,

Coefficient of $x^0=0$

We get $a_3=\frac{a_0}{4}$

Coefficient of $x^1=0$

We get $a_2=\frac{-3a_0}{2}$

Coefficient of $x^2=0$

We get $a_4=0$

Coefficient of $x^3=0$

We get $a_5 = \frac{-a_0}{20}$

Substituting these values in equation (1) ,we get

$Y = a_0 + a_1 x \frac{-3a_0}{2} x^2 + \frac{a_0}{4} x^3 - \frac{a_0}{20} x^5 \ldots.. + \ldots\ldots$

Hence

$Y = a_0 \frac{-3a_0}{2} x^2 + \frac{a_0}{4} x^3 - \frac{a_0}{20} x^5 \ldots.. + a_1 x.. + \ldots\ldots$

$Y = f(x) = a_0 (1 - \frac{3}{2} x^2 + \frac{1}{4} x^3 - \frac{1}{20} x^5 \ldots..) + a_1 x.. + \ldots\ldots$

which is required solution