Answer on Question #72880 – Math – Linear Algebra

Question

Find the basis of $M2 = \{(a, b, c, d): a, b, c, d \in R\}$

Solution

A basis can be defined as a linearly independent subset of vector space of maximal cardinality possible. The number of elements in basis is called the dimension of respective vector space

For n-dimensional vector space R^n its basis is any ordered set of linearly independent vectors $\{e_1, \dots, e_n\}$.

Since $M2 = \{(a, b, c, d): a, b, c, d \in R\}$ defines all 4-tuples of real numbers it's a 4-dimensional real vector space by definition. Hence its basis is a set of 4 vectors $\{e_1(e_{11}, e_{12}, e_{13}, e_{14}), \dots, e_4(e_{41}, e_{42}, e_{43}, e_{44})\}$.

By the invertible matrix theorem, rows of an invertible matrix are linearly independent. Therefore, if the determinant of a matrix, rows of which are vectors $e_1, \dots, e_4$, is not equal to zero, the given vectors form a basis of M2 set:

$$det\begin{bmatrix} e_{11} & \cdots & e_{14} \\ \vdots & \ddots & \vdots \\ e_{41} & \cdots & e_{44} \end{bmatrix} \neq 0 \Rightarrow \qquad \Rightarrow det\begin{bmatrix} e_{11} & \cdots & e_{14} \\ \vdots & \ddots & \vdots \\ e_{41} & \cdots & e_{44} \end{bmatrix} = e_{11} det\begin{bmatrix} e_{22} & e_{23} & e_{24} \\ e_{32} & e_{33} & e_{34} \\ e_{42} & e_{43} & e_{44} \end{bmatrix} - e_{12} det\begin{bmatrix} e_{21} & e_{23} & e_{24} \\ e_{31} & e_{33} & e_{34} \\ e_{41} & e_{43} & e_{44} \end{bmatrix} + e_{13} det\begin{bmatrix} e_{21} & e_{22} & e_{24} \\ e_{31} & e_{32} & e_{34} \\ e_{41} & e_{42} & e_{44} \end{bmatrix} - e_{14} det\begin{bmatrix} e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \end{bmatrix} =$$

$$e_{11}(e_{22}e_{33}e_{44} + e_{23}e_{34}e_{42} + e_{24}e_{32}e_{43} - e_{24}e_{33}e_{42} - e_{23}e_{32}e_{44} - e_{22}e_{34}e_{43}) -$$
$$-e_{12}(e_{21}e_{33}e_{44} + e_{23}e_{34}e_{41} + e_{24}e_{31}e_{43} - e_{24}e_{33}e_{41} - e_{23}e_{31}e_{44} - e_{21}e_{34}e_{43}) +$$
$$+e_{13}(e_{21}e_{32}e_{44} + e_{22}e_{34}e_{41} + e_{24}e_{31}e_{42} - e_{24}e_{32}e_{41} - e_{22}e_{31}e_{44} - e_{21}e_{34}e_{42}) -$$
$$-e_{14}(e_{21}e_{32}e_{43} + e_{22}e_{33}e_{41} + e_{23}e_{31}e_{42} - e_{23}e_{32}e_{41} - e_{22}e_{31}e_{43} - e_{21}e_{33}e_{42}) \neq 0$$

As shown above, the given determinant can be found alphabetically, giving an inequality with sixteen variables. Since solving given inequality to find all possible basis vectors analytically is complex and unneeded because only one basis is required to find, the good way to solve the problem would be setting vectors as rows of a matrix which is known to be invertible.

Using an identity matrix, which is invertible, gives a basis which reflects the rows of given matrix - $\{e_1(1,0,0,0), e_2(0,1,0,0), e_3(0,0,1,0), e_4(0,0,0,1)\}$. This basis is a commonly known standard basis for the R^4 real vector space.

Answer: $\{(1,0,0,0), (0,1,0,0)(0,0,1,0)(0,0,0,1)\}$.

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