Answer on Question #72876 – Math – Functional Analysis

Question

Let d be a metric on a set X. Determine all constant k such that $(k + d)$ is a metric on X. Give a hint or prove it shortly.

Solution

By the definition of a metric, the second condition should be satisfied:

$$x = y \Leftrightarrow (k + d)(x, y) = 0.$$

On the other hand, if $x = y$, then

$$(k + d)(x, y) = k + d(x, y) = k + 0 = k.$$

Therefore, $(k + d)$ is a metric only if $k = 0$. The converse proposition is also true.

Answer:

$k = 0$.