

Answer on Question #72875 – Math – Real Analysis

Question

Prove shortly that $|d(x,z)-d(y,z)|\leq d(x,y)$ Give its Hint or prove it shortly.

Solution

By the triangle inequality $d(x, z) \leq d(x, y) + d(y, z) \rightarrow$

$$\rightarrow d(x, z) - d(y, z) \leq d(x, y). \quad (1)$$

Also, by the triangle inequality $d(y, z) \leq d(y, x) + d(x, z) \rightarrow$

$$\rightarrow -d(y, x) \leq d(x, z) - d(y, z) \text{ or}$$

$$d(x, z) - d(y, z) \geq -d(x, y) \quad (2)$$

because

$$d(x, y) = d(y, x)$$

It follows from (1) and (2) that

$$|d(x, z) - d(y, z)| \leq d(x, y).$$