ANSWER on Question #72783 Math. Complex Analysis

Simplify

$$\int_1^2 \sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt$$

SOLUTION

$$\int_1^2 \sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt = \int_1^2 \sqrt{1+\frac{1}{t^2}+\underbrace{((\ln t)^2+2\ln t+1)}_{(\ln t+1)^2}}\,dt =$$

$$= \int_1^2 \sqrt{1+\frac{1}{t^2}+(1+\ln t)^2}\,dt = \int_1^2 \sqrt{\frac{t^2+1+t^2\cdot(1+\ln t)^2}{t^2}}\,dt =$$

$$= \int_1^2 \frac{\sqrt{t^2+1+t^2\cdot(1+\ln t)^2}}{t}\,dt \equiv \int_1^2 \sqrt{t^2+1+t^2\cdot(1+\ln t)^2}\,\frac{dt}{t} =$$

$$= \begin{bmatrix} \ln t = k \to \dfrac{dt}{t} = dk \\ t = e^k \\ t = 1 \to k = \ln 1 = 0 \\ t = 2 \to k = \ln 2 \end{bmatrix} = \int_0^{\ln 2} \sqrt{(e^k)^2+1+(e^k)^2\cdot(1+k)^2}\,dk =$$

$$= \int_0^{\ln 2} \sqrt{e^{2k}((1+k)^2+1)+1}\,dk$$

ANSWER

$$\int_1^2 \sqrt{1+\frac{1}{t^2}+(\ln t)^2+2\ln t+1}\,dt = \int_0^{\ln 2} \sqrt{e^{2k}((1+k)^2+1)+1}\,dk$$